Inconsistency-Tolerant Reasoning with Classical Logic and Large Databases

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Presenting on joint work with:

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Challenge 1: Inconsistencies

- Occasional errors and disagreements are unavoidable in real-world data.
  - Data acquisition error
  - Out-of-sync
  - Genuine disagreement: Julius Caesar birth year
  - Semantic disagreement: measuring GDP
  - Approximation – apparent contradictions

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Tolerate Inconsistency

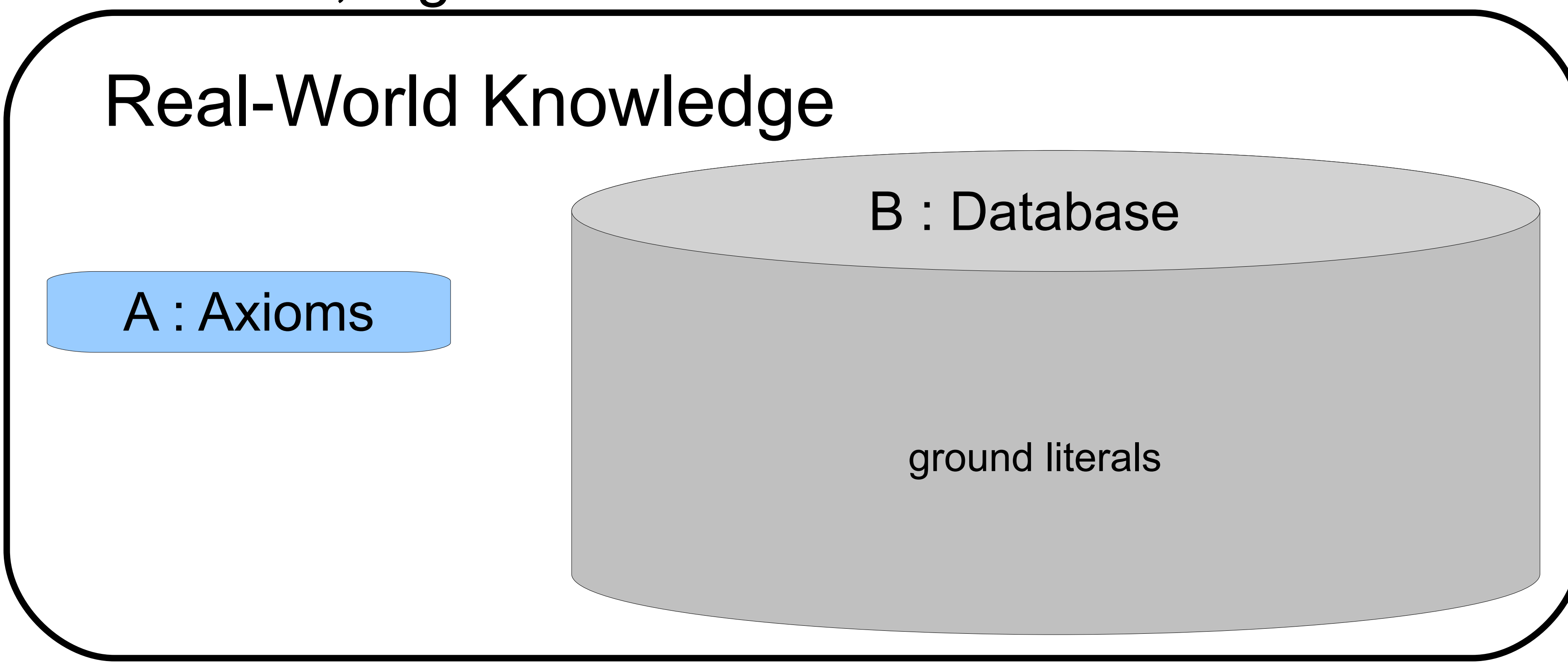
- Classical logic does not tolerate inconsistency
  - If $K \models \bot$ then $K \models \varphi$ for any sentence φ
- Many inconsistency-tolerant reasoning methods
  - Strict Existential Entailment

Challenge 2: Large Premise Set

- Vast amounts of data stored in relational databases
  - 10 Petabytes in Yahoo!'s Everest
- Most automated reasoning systems not designed to handle large premise sets

Real-World Knowledge

- Knowledge in the real world split naturally into
  - Data, represented in databases
  - Axioms, logical sentences



Presentation Outline

- Definition of Strict Existential Entailment
- Naïve method
- Our approach: compilation

Strict Existential Entailment

- Given a set of axioms A and a database B,
  - $A, B \models_E \mathbf{I}(\overline{\mathbf{a}})$

    $\Leftrightarrow$ a consistent portion B* of B classically entails $\mathbf{I}(\overline{\mathbf{a}})$

    – ie. $\exists B^* \sqsubset B \cdot A \cup B^* \not\models \bot$ and $A \cup B^* \models \mathbf{I}(\overline{\mathbf{a}})$

- Strict entailment for short

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Example

- Axioms A:
  - $p(X,Y) \vee \neg q(Z,a) \vee r(Z)$
  - $p(a,U) \vee \neg q(U,a)$
- Database B:
  - $\neg p(a,b)$
  - $q(a,a)$
  - $q(b,a)$

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Example

- Axioms A:
  - p(X,Y) ∨ ¬q(Z,a) ∨ r(Z)
  - p(a,U) ∨ ¬q(U,a)
- Database B:
  - ¬p(a,b) [1]
  - q(a,a)
  - q(b,a)

→

r(a)
¬q(b,a)

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Example

- Axioms A:
  - p(X,Y) ∨ ¬q(Z,a) ∨ r(Z)
  - p(a,U) ∨ ¬q(U,a)
- Database B:
  - ¬p(a,b)
  - q(a,a)
  - q(b,a)

2

r(a)
¬q(b,a)

→

p(a,a)
p(a,b)

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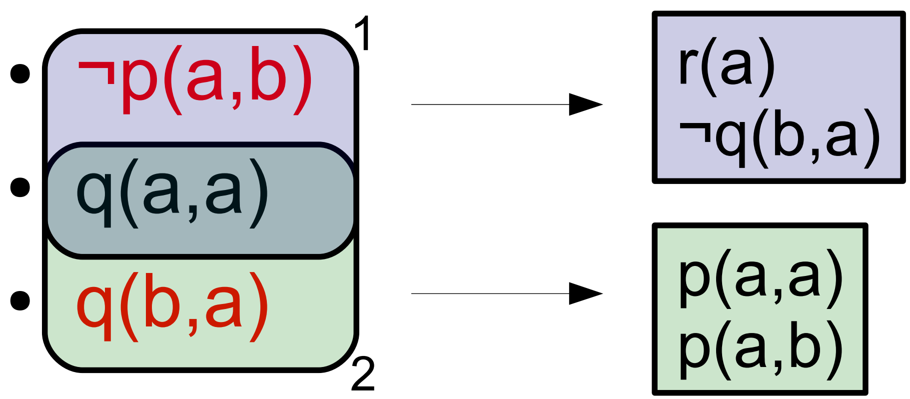
Example

- Axioms A:
  - p(X,Y) ∨ ¬q(Z,a) ∨ r(Z)
  - p(a,U) ∨ ¬q(U,a)
- Database B:



- r(b) is excluded!

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Naïve Method

- Consider each consistent (maximal) subset of the data
- Find the the classically entailed conclusions for each subset
- There may be exponentially many consistent maximal subsets!

p(A,B)

A	a1	a1	a2	a2	...	an	an
B	b0	b1	b0	b1	...	b0	b1

Axiom: $p(X,Y) \wedge p(X,Z) \rightarrow Y = Z$

A relation of **2n** tuples has **2^n** consistent maximal portions!

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Concentrate on the Axioms

- Axioms A:
  - p(X,Y) ∨ ¬q(Z,a) ∨ r(Z)
  - p(a,U) ∨ ¬q(U,a)
- Database B:
  - ¬p(a,b)
  - q(a,a)
  - q(b,a)
- Deduction:
  - `¬p(a,b)      q(b,a)      r(Z)`
  - `p(a,U)      ¬q(U,a)`

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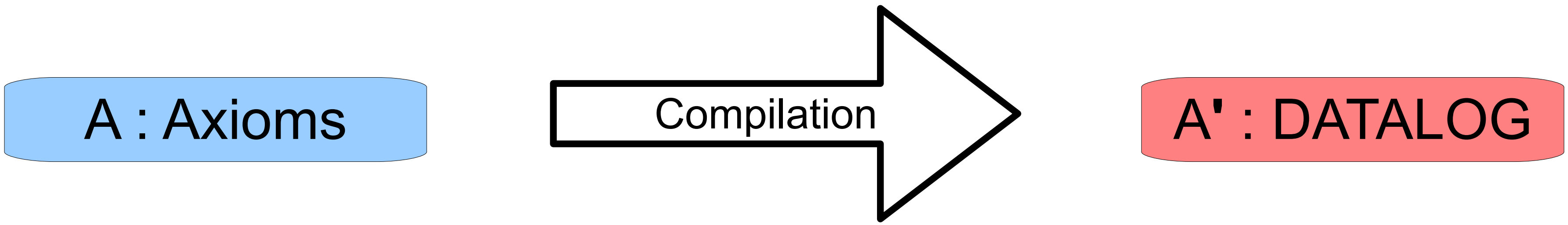
Inconsistency-Tolerant Compilation Approach



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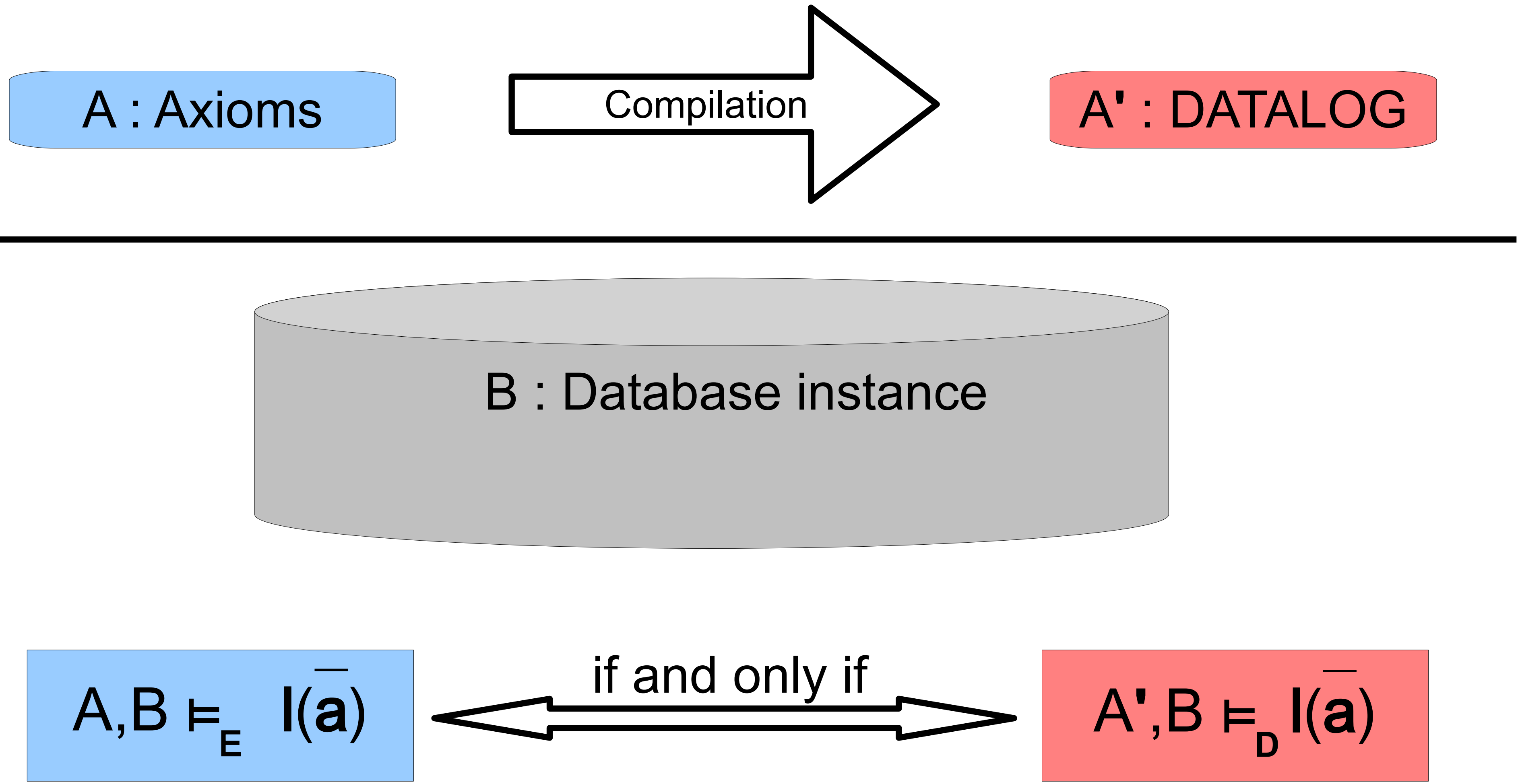
Inconsistency-Tolerant Compilation Approach



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Setting

- Axioms A: first-order logic with equality:
  - Function-free
  - Universal clause
- Relational database B
- Domain closure assumption
- Unique names assumption

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Compilation to DATALOG

p(X,Y) ¬q(Z,a) r(Z)
p(a,U) ¬q(U,a)

*See Algorithm 1 in paper

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Compilation to DATALOG

$p(X,Y) \quad \neg q(Z,a) \quad r(Z)$

$p(a,U) \quad \neg q(U,a)$

resolution

contrapositives

$$
\begin{aligned}
p(X,Y) &\leftarrow q(Z,a) \wedge \neg r(Z) \\
\neg q(Z,a) &\leftarrow p(X,Y) \wedge \neg r(Z) \\
r(Z) &\leftarrow \neg p(X,Y) \wedge q(Z,a) \\
\neg q(U,a) &\leftarrow \neg p(a,U) \\
p(a,U) &\leftarrow q(U,a)
\end{aligned}
$$

*See Algorithm 1 in paper

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Compilation to DATALOG

$$\begin{array}{lll} p(X,Y) & \neg q(Z,a) & r(Z) \\ p(a,U) & \neg q(U,a) & \end{array}$$

resolution

contrapositives

$$\begin{array}{rcllll} p(X,Y) & \leftarrow & q(Z,a) & \wedge & \neg r(Z) \\ \neg q(Z,a) & \leftarrow & p(X,Y) & \wedge & \neg r(Z) \\ r(Z) & \leftarrow & \neg p(X,Y) & \wedge & q(Z,a) \\ \neg q(U,a) & \leftarrow & \neg p(a,U) & & \\ p(a,U) & \leftarrow & q(U,a) & & \end{array}$$

DATALOG

$$\begin{array}{lcllll} p^{+}(X,Y) & :- & q(Z,a) & \wedge & \neg r(Z) \\ q^{-}(Z,a) & :- & p(X,Y) & \wedge & \neg r(Z) \\ r^{+}(Z) & :- & \neg p(X,Y) & \wedge & q(Z,a) \\ q^{-}(U,a) & :- & \neg p(a,U) & & \\ p^{+}(a,U) & :- & q(U,a) & & \end{array}$$

*See Algorithm 1 in paper

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Inconsistency

- Database B:
  - ¬p(a,b)
  - q(a,a)
  - q(b,a)

```
r+(Z) :- ¬p(X,Y) ∧ q(Z,a)
```

```
r+(a) :- ¬p(a,b) ∧ q(a,a)
```

```
r+(b) :- ¬p(a,b) ∧ q(b,a)
```

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Inconsistency

- Database B:
  - ¬p(a,b)
  - q(a,a)
  - q(b,a)

$r^{+}(Z)$:- ¬p(X,Y) ∧ q(Z,a)

$r^{+}(a)$:- ¬p(a,b) ∧ q(a,a)

~~r^{+}~~(b) :- ¬p(a,b) ∧ q(b,a)

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Augment for Inconsistency

- rule: `r⁺(Z) :- ¬p(X,Y) ∧ q(Z,a)`
- Negated rule body

  ¬b : p(X,Y) ∨ ¬q(Z,a)
- Axiom clause c : p(a,U) ∨ ¬q(U,V)

*See Algorithm 3 in paper

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Augment for Inconsistency

- rule: `r⁺(Z) :- ¬p(X,Y) ∧ q(Z,a)`
- Rule body

  $b: \neg p(X,Y) \vee q(Z,a)$
- Axiom clause $c: p(a,U) \vee \neg q(U,V)$
- $c, b\sigma \models \bot \Leftrightarrow [X = a \wedge Y = Z]\sigma$
- Augmented rule:

  `r⁺(Z) :- ¬p(X,Y) ∧ q(Z,a) ∧ ¬[X = a ∧ Y = Z]`

*See Algorithm 3 in paper

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Evaluate on Example Data

- Database B:
  - ¬p(a,b)
  - q(a,a)
  - q(b,a)

$r^+(Z)$:- ¬p(X,Y) ∧ q(Z,a) ∧ ¬[X = a ∧ Y = Z]

$r^+(a)$:- ¬p(a,b) ∧ q(a,a) ∧ ¬[a = a ∧ b = a]

$r^+(b)$:- ¬p(a,b) ∧ q(b,a) ∧ ¬[a = a ∧ b = b]

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Termination

- The compilation algorithm terminates when the input axioms A has a finite closure under resolution and factoring.

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Sound and Complete

- Theorem:
  - Assume:
    - Function-free universal axioms in FOL with =
    - Domain closure assumption
    - Unique names assumption
  - The compilation is sound and complete for strict existential entailment.

$$A,B \models_E I(\bar{a}) \quad \text{if and only if} \quad A',B \models_D I(\bar{a})$$

*See Theorems 1 and 2 in paper

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Features

- Compile independently of data
- flat Datalog$^\neg$ ⊏ RA ⊏ SQL
  - Polynomial data complexity
- Simple layer over existing DBMS
  - Custom code ignores data
  - Low cost of adoption
  - Leverage current state-of-the-art infrastructure
- Reuse on different/evolving data

Related Work

- Inconsistency tolerance based on classical logic
  - (Hunter 1998; Besnard & Hunter 2005; Konieczny, Lang & Marquis 2005; Huang, van Harmelen & ten Teije 2005; Zamansky & Avron 2006; Flouris et al. 2006; Subrahmanian & Amgoud 2007; Hunter and Konieczny 2008; Everaere, Konieczny, and Marquis 2008; Besnard and Hunter 2008)
- Knowledge compilation
  - (Darwiche & Marquis 2002; Selman & Kautz 1996; Nagy, Lukacsy & Szeredi 2006; Calvanese et al. 2008; Besnard & Hunter 2006; Hinrichs & Genesereth 2008; Cadoli & Mancini 2002; Nagy, Lukacsy & Szeredi 2006; Calvanese et al. 2008; Flouris et al. 2006; Huang, van Harmelen & ten Teije 2005; Gomez, Chesnevar & Simari 2008)

Future Work

- Existential quantification
- When resolution takes long:
  - compile into recursive Datalog or Prolog
- Give relationship between conclusions
  - rebuttal and undercutting

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