Logic in Secondary School Education

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1. Introduction

Logic is one of the oldest intellectual disciplines in human history. It dates back to Aristotle. It has been studied through the centuries by people like Leibniz, Boole, Russell, Turing, and many others. And it is still a subject of active investigation today.

We use Logic in just about everything we do. We use it in our professional discussions. We use it in our personal conversations. We use the language of Logic to state observations, to define concepts, and to formalize theories. We use logical reasoning to derive conclusions from these bits of information. We use logical proofs to convince others of our conclusions.

Logic is essential for many STEM disciplines, especially computer science. We know that calculus is important to physics, and we teach is at the high school level. Well, logic is the language of computer science and computer science is arguably getting to be as important as physics.

More broadly though, Logic is useful for everyone. It is more relatable than traditional quantitative math for many students, students who see relationships between people and things but who are uncomfortable reducing everything to numbers.

Advertisers, politicians, companies, organizations, friends, family, and experts want us to buy their products, vote for them, or support what they believe and want to do. Logic helps us spot the hype, the nonsense, who is wrong and right.

And we are not alone! Logic is increasingly being used by computers - to prove mathematical theorems, to validate engineering designs, to diagnose failures, to encode and analyze laws and regulations and business rules. Logic is also becoming more common at the interface between man and machine, in "logic-enabled" computer systems, where users can view and edit logical sentences. And Logic is sometimes used not just by users in communicating with computer systems but by software engineers in building those systems (using a programming methodology known as *logic programming*).

The importance of Logic in our lives raises the question of how we come by the ability to use Logic. To some extent, it is innate, like our ability to recognize faces. However, some elements of Logic need to be taught explicitly, in the same way that we teach algebra. And this raises the question of how and when this education should take place.

The ancient Greeks thought it sufficiently important that Logic was one of the three subjects in the Greek Trivium, along with Grammar and Rhetoric. Oddly, this is not the case in the American educational system. Logic occupies a relatively small place in the modern curriculum.

Admittedly, some elements of logic do appear in secondary school courses today, e.g. elementary proofs in geometry, discussions of fallacies in writing courses, and of course tips and techniques for using search engines and other computer systems. However, Logic is not taught as a standalone topic in most secondary schools today. Imagine saying we do not need to teach arithmetic because some of the elements are covered in chemistry and history.

It is our belief that logic is important enough to deserve treatment as a standalone topic There is easily enough material for a standalone course. Topics in Boolean Logic include logical connectives (e.g. and, or, not), contrapositives, converses, inverses, de Morgan's Laws, counterfactual statements, truth tables, and propositional proofs. Topics in Relational Logic include variables and quantifiers, model checking, and relational proofs. More general topics include negation as failure (knowing not versus not knowing), the differences between consistency and entailment and equivalence, and fallacies and paradoxes.

In this article, we discuss the the issues involved in creating such a course and getting it adopted at the secondary school level. We start by arguing for a focus on Symbolic Logic. We talk about an approach to teaching this material that is rigorous yet still accessible at the secondary school level. And we conclude with a call to action and an approach for introducing Logic as a standalone course in secondary schools.

2. Symbolic Logic

Although it is possible to teach Logic using only English language, this is problematic. Natural language sentences can be complex; they can be ambiguous; and failing to understand the meaning of a sentence can lead to errors in reasoning.

Even very simple sentences can be troublesome. Here we see two grammatically legal sentences. They are the same in all but the last word, but their structure is entirely different. In the first, the main verb is *blossoms*, while in the second *blossoms* is a noun and the main verb is *sank*.

The cherry blossoms in the Spring.
The cherry blossoms in the Spring sank.

As another example of grammatical complexity, consider the following excerpt taken from the University of Michigan lease agreement. The sentence in this case is sufficiently long and the grammatical structure sufficiently complex that people must often read it several times to understand precisely what it says.

> *The University may terminate this lease when the Lessee, having made application and executed this lease in advance of enrollment, is not eligible to enroll or fails to enroll in the University or leaves the University at any time prior to the expiration of this lease, or for violation of any provisions of this lease, or for violation of any University regulation relative to resident Halls, or for health reasons, by providing the student with written notice of this termination 30 days prior to the effective data of termination, unless life, limb, or property would be jeopardized, the Lessee engages in the sales of purchase of controlled substances in violation of federal, state or local law, or the Lessee is no longer enrolled as a student, or the Lessee engages in the use or possession of firearms, explosives, inflammable liquids, fireworks, or other dangerous weapons within the building, or turns in a false alarm, in which cases a maximum of 24 hours notice would be sufficient.*

As an example of ambiguity, suppose I were to write the sentence *There's a girl in the room with a* *telescope*. See Figure 6 for two possible meanings of this sentence. Am I saying that there is a girl in a room containing a telescope? Or am I saying that there is a girl in the room and she is holding a telescope?

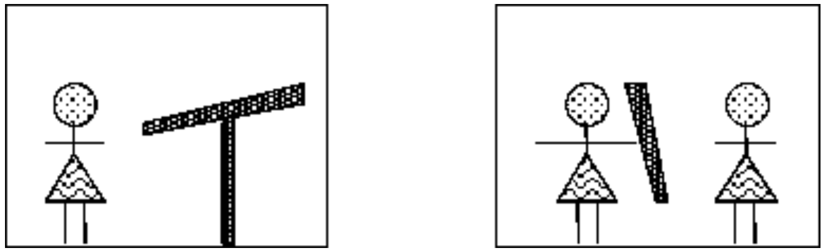

Figure 6 - *There's a girl in the room with a telescope*.

Such complexities and ambiguities can sometimes be humorous if they lead to interpretations the author did not intend. See the examples below for some infamous newspaper headlines with multiple interpretations. Using a formal language eliminates such unintentional ambiguities (and, for better or worse, avoids any unintentional humor as well).

Crowds Rushing to See Pope Trample 6 to Death
Journal Star, Peoria, 1980

Scientists Grow Frog Eyes and Ears
The Daily Camera, Boulder, 2000

British Left Waffles On Falkland Islands

Food Stamp Recipients Turn to Plastic
The Miami Herald, 1991

Indian Ocean Talks
The Plain Dealer, 1977

Fried Chicken Cooked in Microwave Wins Trip
The Oregonian, Portland, 1981

As an illustration of errors that arise in reasoning with sentences in natural language, consider the following examples. In the first, we use the transitivity of the *better* relation to derive a conclusion about the relative quality of champagne and soda from the relative quality of champagne and beer and the relative quality or beer and soda. So far so good.

Champagne is better than beer.
Beer is better than soda.
Therefore, champagne is better than soda.

Now, consider what happens when we apply the same transitivity rule in the case illustrated below. The form of the argument is the same as before, but the conclusion is somewhat less believable. The problem in this case is that the use of *nothing* here is syntactically similar to the use of *beer* in the preceding example, but in English it means something entirely different.

Bad sex is better than nothing.
Nothing is better than good sex.
Therefore, bad sex is better than good sex.

Symbolic Logic eliminates these difficulties through the use of a formal language for encoding information. Given the syntax and semantics of this formal language, we can give a precise definition for the notion of logical conclusion. Moreover, we can establish precise reasoning rules that produce all and only logical conclusions.

In this regard, there is a strong analogy between the methods of Formal Logic and those of high school algebra. To illustrate this analogy, consider the following algebra problem.

> *Xavier is three times as old as Yolanda. Xavier's age and Yolanda's age add up to twelve. How old are Xavier and Yolanda?*

Typically, the first step in solving such a problem is to express the information in the form of equations. If we let x represent the age of Xavier and y represent the age of Yolanda, we can capture the essential information of the problem as shown below.

$$x - 3y = 0$$
$$x + y = 12$$

Using the methods of algebra, we can then manipulate these expressions to solve the problem. First we subtract the second equation from the first.

$$\begin{array}{l} x - 3y = 0 \\ \underline{x + y = 12} \\ -4y = -12 \end{array}$$

Next, we divide each side of the resulting equation by -4 to get a value for y. Then substituting back into one of the preceding equations, we get a value for x.

$$x = 9$$
$$y = 3$$

Now, consider the following logic problem.

> *If Mary loves Pat, then Mary loves Quincy. If it is Monday and raining, then Mary loves Pat or Quincy. If it is Monday and raining, does Mary love Quincy?*

As with the algebra problem, the first step is formalization. Let p represent the possibility that Mary loves Pat; let q represent the possibility that Mary loves Quincy; let m represent the possibility that it is Monday; and let r represent the possibility that it is raining.

With these abbreviations, we can represent the essential information of this problem with the following logical sentences. The first says that *p implies q*, i.e. if Mary loves Pat, then Mary loves Quincy. The second says that *m and r implies p or q*, i.e. if it is Monday and raining, then Mary loves Pat or Mary loves Quincy.

$$p \Rightarrow q$$
$$m \wedge r \Rightarrow p \vee q$$

As with Algebra, Formal Logic defines certain operations that we can use to manipulate expressions. The operation shown below is a variant of what is called *Propositional Resolution*.

The expressions above the line are the premises of the rule, and the expression below is the conclusion.

$$\frac{\begin{array}{ll} p_1 \wedge \ldots \wedge p_k & \Rightarrow q_1 \vee \ldots \vee q_l \\ r_1 \wedge \ldots \wedge r_m & \Rightarrow s_1 \vee \ldots \vee s_n \end{array}}{p_1 \wedge \ldots \wedge p_k \wedge r_1 \wedge \ldots \wedge r_m \Rightarrow q_1 \vee \ldots \vee q_l \vee s_1 \vee \ldots \vee s_n}$$

There are two elaborations of this operation. (1) If a proposition on the left hand side of one sentence is the same as a proposition on the right hand side of the other sentence, it is okay to drop the two symbols, with the proviso that *only one* such pair may be dropped. (2) If a constant is repeated on the same side of a single sentence, all but one of the occurrences can be deleted.

We can use this operation to solve the problem of Mary's love life. Looking at the two premises above, we notice that *p* occurs on the left-hand side of one sentence and the right-hand side of the other. Consequently, we can cancel the *p* and thereby derive the conclusion that, if is Monday and raining, then Mary loves Quincy or Mary loves Quincy.

$$\frac{\begin{array}{c} p \Rightarrow q \\ m \wedge r \Rightarrow p \vee q \end{array}}{m \wedge r \Rightarrow q \vee q}$$

Dropping the repeated symbol on the right hand side, we arrive at the conclusion that, if it is Monday and raining, then Mary loves Quincy.

$$\frac{m \wedge r \Rightarrow q \vee q}{m \wedge r \Rightarrow q}$$

This example is interesting in that it showcases our formal language for encoding logical information. As with algebra, we use symbols to represent relevant aspects of the world in question, and we use operators to connect these symbols in order to express information about the things those symbols represent.

The example also introduces one of the most important operations in Formal Logic, viz. Resolution (in this case a restricted form of Resolution). Resolution has the property of being *complete* for an important class of logic problems, i.e. it is the *only* operation necessary to solve any problem in the class.

3. Logic Course

In order to facilitate the introduction of Logic into schools, we have put together a course we think is suitable for this purpose - the Stanford Introduction to Logic.

The approach to teaching Logic used in this course emerged from more than 10 years of experience in teaching the logical foundations of Artificial Intelligence and more than 20 years of experience in teaching Logic for Computer Scientists. The result of this experience is an approach that differs from that taken by other books in Logic in two essential ways, one having to do with content, the other with technology.

The primary difference in content concerns that semantics of the logic that is taught. Like many other courses on Logic, our course covers first-order syntax and first-order proof theory plus induction. However, unlike other courses, our course starts with Herbrand semantics rather than the more traditional Tarskian semantics.

In Tarskian semantics, we define an interpretation as a universe of discourse together with a function (1) that maps the object constants of our language to objects in a universe of discourse and (2) that maps relation constants to relations on that universe. We define variable assignments as assignments to variables. We define the semantics of quantified expressions as variations on variable assignments, saying, for example, that a universally quantified sentence is true for a given interpretation if and only if it is true for every variation of the given variable assignment. It is a mouthful to say and even harder for students to understand.

In Herbrand semantics, we start with the object constants, function constants, and relation constants of our language; we define the Herbrand base (i.e. the set of all ground atoms that can be formed from these components); and we define a model to be an arbitrary subset of the Herbrand base. That is all. In Herbrand semantics, an arbitrary logical sentence is logically equivalent to the set of all of its instances. A universally quantified sentence is true if and only if all of its instances are true. There are no interpretations and no variable assignments and no variations of variable assignments.

Although both approaches ultimately end up with the same deductive mechanism, we get there in two different ways. Deciding to use Herbrand semantics was not an easy to choice to make. It took years to get the material right and, even then, it took years to use it in teaching Logic. Although there are some slight disadvantages to this approach, experience suggests that the advantages significantly outweigh those disadvantages. This approach is considerably easier for students to understand and leaves them with a deeper understanding of what Logic is all about.

The second major difference between our approach and that of others is the substantial use of educational technology. In addition to the text of the book in print and online, there are online exercises (with automated grading), a hyperlinked glossary with references to related text and exercises), online Logic tools and applications, online videos of lectures, and an online forum for discussion.

4. Conclusion

In summary, we believe that whether or not to teach Logic in high school is not really a point of debate. We need to make this education available in high schools all across America. Logic has to be and must be taught to all students if we want to prepare responsible citizens. A logically literate populace will know how to ask the right questions of their leaders, how to spot fallacies, and most importantly, how to make decisions that truly align with their values. Logically fluent citizenry is not really an option for any functional democracy, and there is so much at stake in thinking systematically that training for it must be included in the curriculum and cannot be left to chance. With a background in Logic, students will be well-prepared for careers in STEM that are so critical for any country to be competitive in the modern day world. The goal of making Logic education available across the country may sound formidable; but it is achievable by mobilizing students and parents to demand for it and empowering our high school teachers to teach it.