Logic Programming
Datalog

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Optimizing Hex

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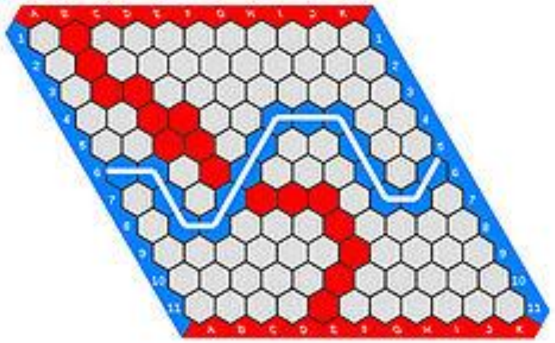
What is Hex?

- Hex is a two-player game invented by John Nash and Piet Hein (independently).
- Players take turns placing tiles on any cell of their choosing.
- Players win by connecting a chain of tiles, such that they form a line spanning from one edge of the board to the opposite edge.
- Hex is a game commonly studied by Mathematicians in Computer Science in order to shed light on topics including: graph theory, combinatorics, game theory, and AI.
- In 11×11 Hex, there are approximately 2.4×10^{56} possible legal positions! (Approximated using an exponential function and branching factor analysis)



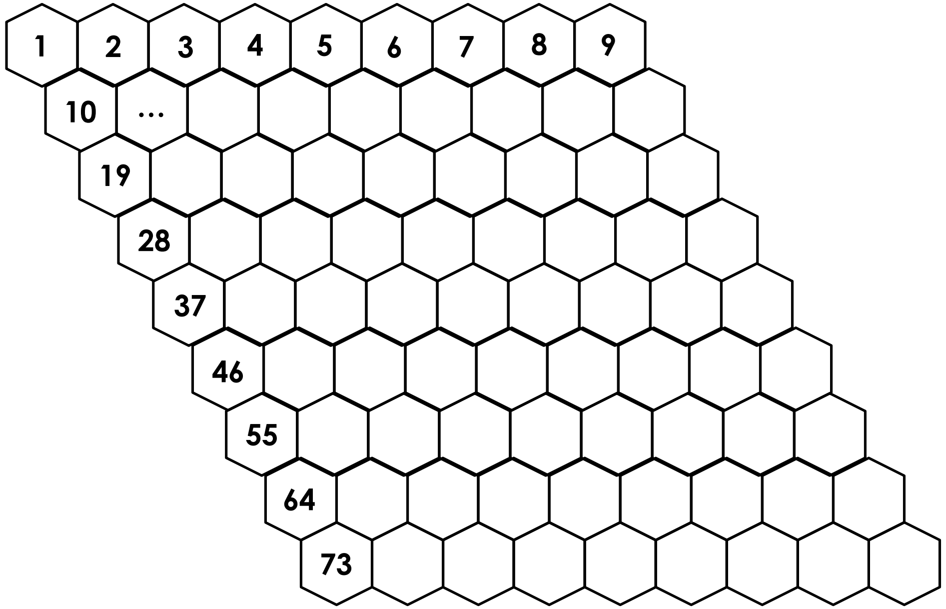
What are we Investigating?

- *What are the different paradigms in which we can encode the rules of Hex?*
- *How does each paradigm perform (relatively)?*

Why Logic Programming? (GDL)

- ***Testing Games in a Generalizable Fashion:*** Logic Programming is the methodology of describing games in the field General Gameplaying. GDL is widely accepted as the language of General Gameplaying!
- ***Condition Testing:*** We are really just solving a condition problem, namely: given this set of data, is X true? Logic Programming is very good for that!
- ***Avoiding the "background implementations":*** In a traditional imperative programming language, we would have to focus on building the "back-end" framework from game-to-game; logic programming avoids that!

General Observations:

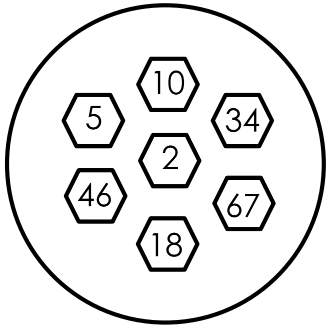


- We can assign each cell in the Hex board a numerical index.
- In this way, we can codify mathematical rules defining adjacency:
  - E.g. Cells X & Y are adjacent if Y = X + 1
- One tile must be in each column (or row) in order for a player to have won (as a *necessary*, but **not sufficient** condition)

Approach #1: Naïve Implementation

- What if you abstracted half of the problem away from logic programming?
- Use logic programming as a “verifier” and another language (Python) to generate the “Winning Sets”.
  - E.g. $\{1,2,3,4,5,6,7,8,9\} \in \mathcal{W}$
- After every move, check if the cells “controlled” by a given player is a superset of the winning set.
  - $\exists w \in \mathcal{W}.\ w \subseteq M$, where M is the set of cells p has played.

Approach #2: Power-set Constraints



1. Maintain set of all cells controlled by player *p*

{28, 2, 12, 13, 42, 16, 8, 45}

…

{55, 11, 3, 31, 21, 42, 8, 27}

2. Generate all* 9-length subsets s.t. each E_i (element in the *i*th position) is a member of the *ith column*

73

$n - 8 \longrightarrow 65 \in S$?

$n + 1 \longrightarrow 74 \in S$?

3. For each element in a set, check if the subsequent element obeys an "adjacency" rule.

To avoid repeatedly checking non-winning sets, one can preserve* ***all* *previous non-winning sets and check all new sets generated by* ***replacing*** *the corresponding column entry in the previously generated sets*

E.g. If you play in Column 6 …

$\{C_{Old}^{(6)} \rightarrow C_{New}^{(6)}\}$ in all sets

Power-set Constraints: Worst-Case Analysis

{28, 2, 12, 13, 42, 16, 8, 45}

…

{55, 11, 3, 31, 21, 42, 8, 27}

2. Generate all 9-length subsets s.t. each E_i (element in the ith position) is a member of the *ith column*

Note:

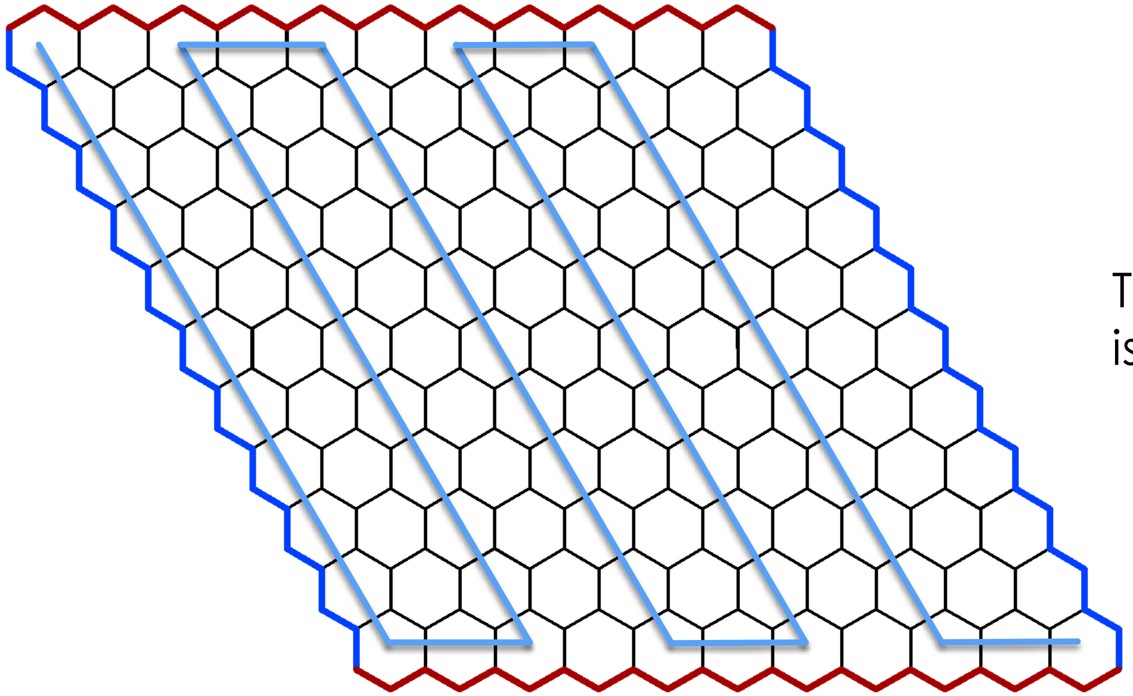
What if we generated *all 9-length* subsets **without** our unique column restriction?

Suppose player *p* controls *n* cells *(n_{max} = 81):*

$$\binom{81}{9} \text{ vs. } 9^9 \longrightarrow \frac{260887834350}{387420489}$$

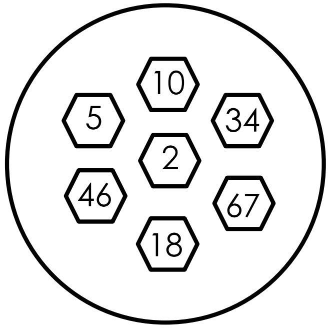
≈ x674 more computations!

But wait! It isn't that simple!



This winning sequence is 61 tiles long!

Approach #2: Power-set Constraints*



1. Maintain set of all cells controlled by player *p*

{28, 2, 12, 13, 42, 16, 8, 45}

…

{55, 11, 3, 31, 21, 42, 8, 27}

2. Generate all* ~~9-length subsets~~ n-length subsets [9, 61], s.t. each E_i (element in the *ith* position) is a member of the *ith column*

73

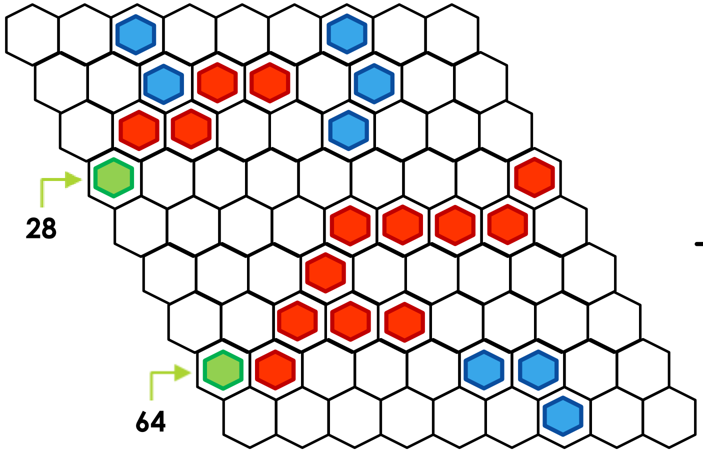
$n - 8 \longrightarrow 65 \in S$?

$n + 1 \longrightarrow 74 \in S$?

$n - 9$?

3. For each element in a set, check if the subsequent element obeys an "adjacency" rule.

Approach #3: Following the Line



Column 1

64

Column 2

$n-8 \longrightarrow 56$ ✗

$n+1 \longrightarrow 65$ ✓

$n+10 \longrightarrow 74$ ✗

Column 3

$n-8 \longrightarrow 57$

$n+1 \longrightarrow 66 \quad \cdots$

$n+10 \longrightarrow 76$

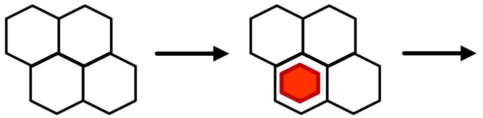
1. For a given player *p*, consider each cell they control in column 1 (Indices: 1, 10, 19, … ,73)

2. Using the adjacency rules, compute all in the next column (column *i + 1)* that would be adjacent to the current cell (in column *i)*.

3. If player *p* controls any of the adjacent cells, repeat the adjacency check. If you can "follow the line" all the way to the end column, the player has won!

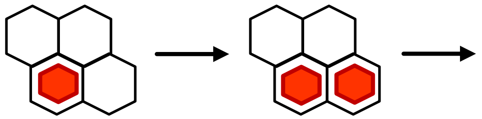
Approach #4 : Minimal Spanning Tree

1. Define a "connected" relation: connected(TREE_NUM, ROW, COL)
2. After each turn, update the connected relations in the dataset:



Lambda:

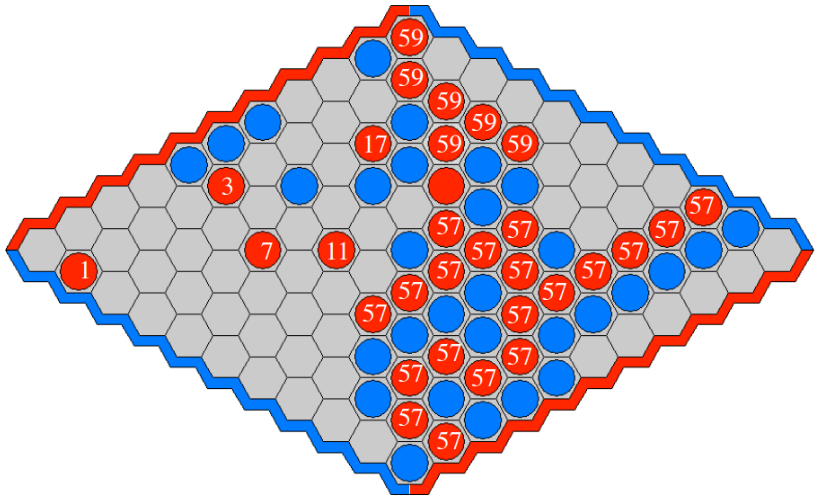
```
connected(1,2,1)
```



Lambda:

```
connected(2,2,1)
connected(2,2,2)
```

3. The game is won if there is some set of connected relations s.t. there exists some "**connected(N_{WIN}, R_1, C)**" and "**connected(N_{WIN}, R_8, C)**" (with analogous reasoning extending to spanning a column). This set of connected relations defines the eponymous *minimal spanning tree*



Credit: M. Genersereth, CS 151, Lecture 12 *(Optimization)*

Beyond the Paradigm: General Optimization Techniques (GOT)

Grounding:

Lambda:

```
p(X) :- index(X)
index(1)
index(2)
index(3)
----------------
p(1) :- index(1)
p(2) :- index(2)
p(3) :- index(3)
index(1)
index(2)
index(3)
```

Sub-goal Reordering:

```
s(X,Y) :- p(X) & r(X,Y) & q(X)
```

↓

```
s(X,Y) :- p(X) & q(X) & r(X,Y)
```

Sub-goal Pruning:

```
r(X,Y) :- p(X,Y) & q(Y) & q(Z)
```

↓

```
r(X,Y) :- p(X,Y) & q(Y)
```

GOT Efficiency Analysis?:

- ***Conjecture:*** The majority of the time is spent in verifying whether a victory exists or not.
- ***Technique:*** Devise a particularly difficult example, and see if the verifier can(not) detect a victory.
  - Analysis was conducted on a board with 34 tiles filled, and no victory determined.

Javascript:

```
grindem(compfinds(read('winner(X)'),read('winner(X)'),repository,library))
```

Eval

Output:
127448 milliseconds

```
winner(red)
```

Without grounding and sub-goal reordering

Javascript:

```
grindem(compfinds(read('winner(X)'),read('winner(X)'),repository,library))
```

Eval

Output:
16 milliseconds

```
winner(red)
```

With grounding and sub-goal reordering

Hex as a Maker-Breaker Game

- A “Maker-Breaker” game can be thought of a game with two distinct players:
  - Maker: wins by taking elements from a finite set until they have a winning set
  - Breaker: wins by stopping the Maker
- Framing Hex as a Maker-Breaker game:
  - **Don’t** think: “Has Red won? Has Blue won?”
  - Think: “Has Red won? Has Red *lost*? (Can Red still win?)”
- Hex implementation:
  - After each play, populate all blank cells with red tiles
  - On Blue’s turn, if a red path still exists, then Red hasn’t lost
  - On Red’s turn, if a red path still exists, then Red can still win!
- Maker-Breaker general strategy: populating available moves with Maker’s moves

Questions?