Introduction to Logic

Michael Genesereth and Eric Kao
Stanford University

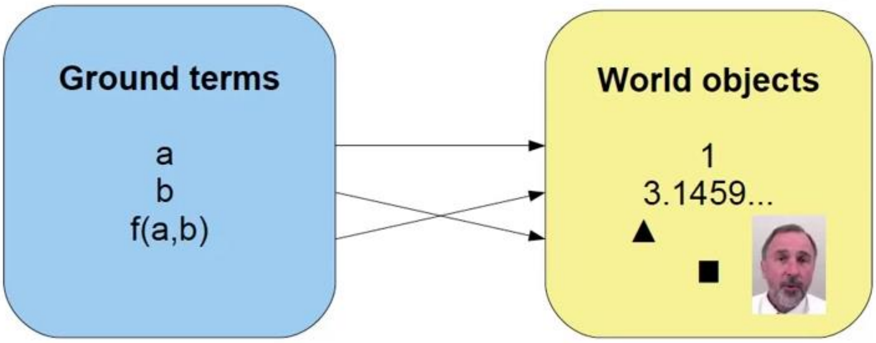
Relational Logic & First-order Logic

Relational Logic (RL)

- Possible worlds are considered over ground terms in the language.
- Over the language <a,b,p>, there are four possible worlds:
  - p(a) false
    p(b) false
  - p(a) true
    p(b) false
  - p(a) false
    p(b) true
  - p(a) true
    p(b) true

First-Order Logic (FOL)

- Ground terms in the language refer to a set of objects in the world.
- Possible worlds are considered over the set of objects.



First-Order Logic (FOL)

- Over the same language <a,b,p>, FOL possible worlds include:
  - Objects: ▲, ■
    a → ▲
    b → ■
    p *true* for ▲, ■
  - Objects: the real numbers
    a → 1
    b → 3.14159...
    p *true* for all rational numbers
    p *false* for all irrational numbers
  - Objects: ▲
    a → ▲
    b → ▲
    p true for ▲
  - There are infinitely many possible worlds

Semantic

- RL semantics is very simple
  - World is a truth assignment on ground sentences
  - Just like a database
- FOL semantic is much more difficult to define and to understand
- http://en.wikipedia.org/wiki/First-order_logic

Describing worlds

- RL more precise in describing worlds
- Over the language L = <a,b,p>

consider Σ = {p(a), p(b)}.

- Exactly one RL world that satisfies Σ.
  - p true for everything

Describing worlds

- but there are many different FOL worlds satisfying Σ
  - Objects: ▲
    a → ▲
    b → ▲
    p true for ▲
- Objects: ▲, ■
  a → ▲
  b → ■
  p *true* for ▲, ■
- Objects: ▲, ■, ☺
  a → ▲
  b → ■
  p *true* for ▲, ■
  p *false* for ☺
- Many more

Describing worlds

- One practical consequence of RL's precision in describing worlds:
  Natural numbers with addition and multiplication
  - Finitely described in RL
  - Impossible in FOL

Deductive power

- FOL deduction is "weaker" than RL
- More entailed conclusions under RL
- $\{p(a), p(b)\} \models_{RL} \forall x.p(x)$
- $\{p(a), p(b)\} \not\models_{FOL} \forall x.p(x)$
  - Objects: ▲, ■, ☺
    a → ▲
    b → ■
    p *true* for ▲, ■
    p *false* for ☺

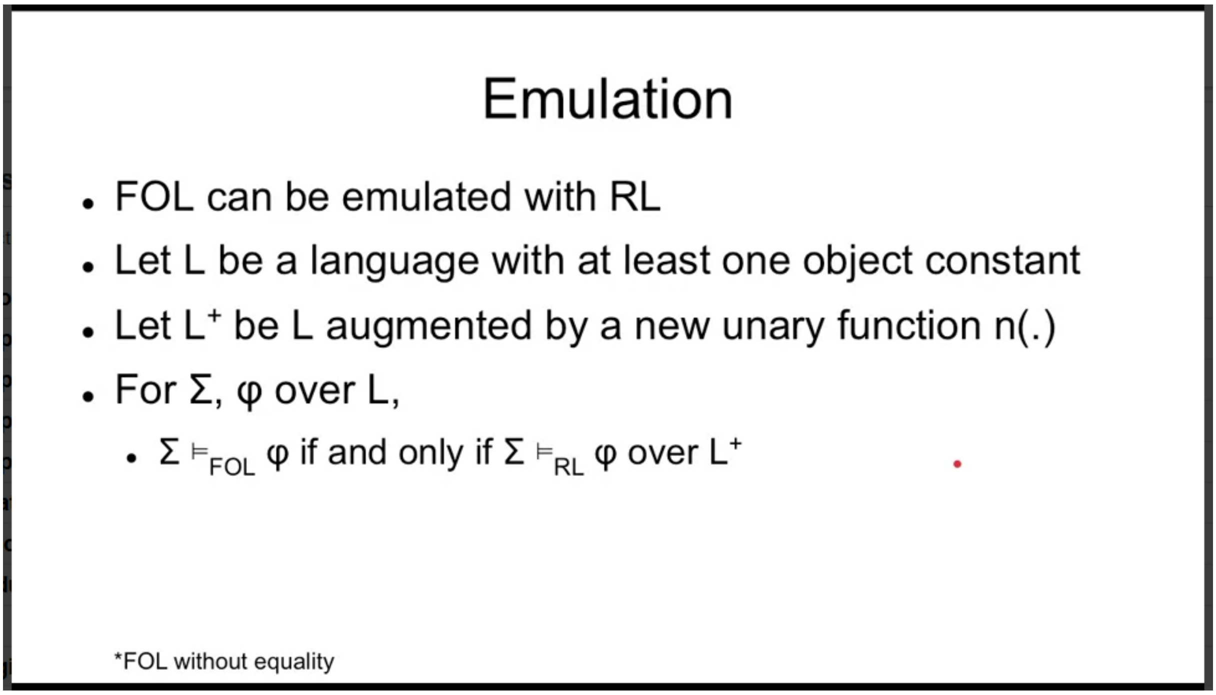
FOL Proof systems

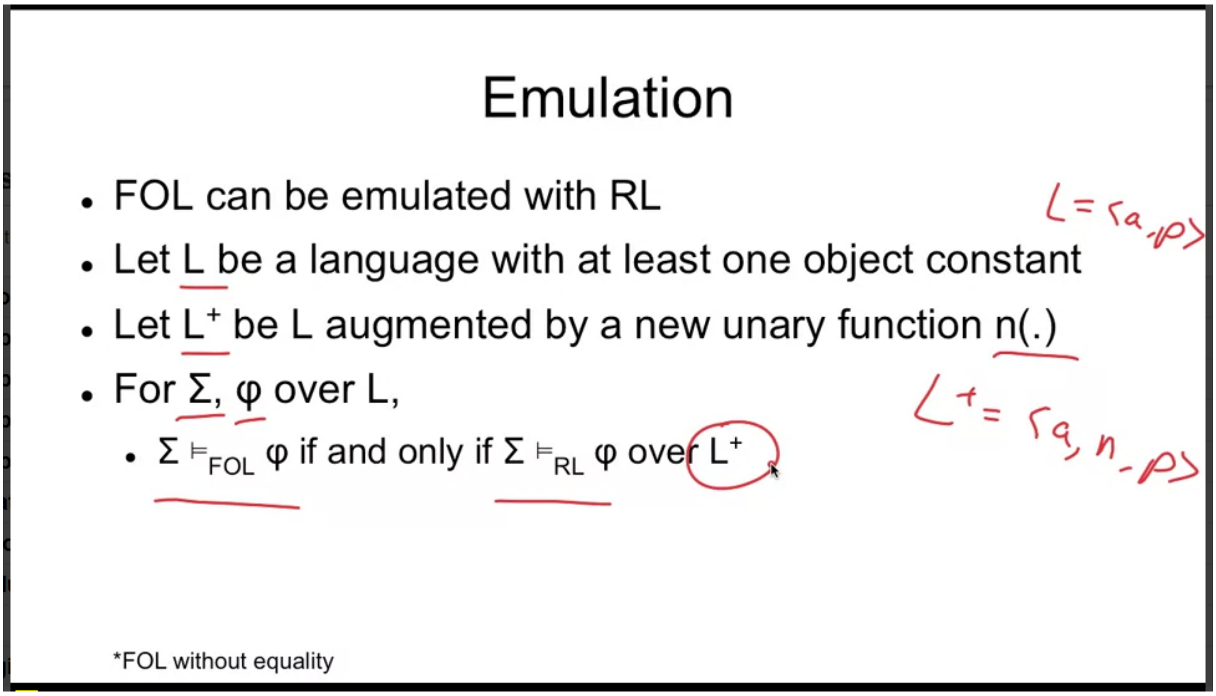
- RL is more powerful than FOL in describing the world.
- What about proving consequences?
- Fitch system without DC, IND is sound and complete for FOL
- $\Sigma \vdash_{Fitch-} \varphi$ iff $\Sigma \models_{FOL} \varphi$

*FOL without equality

RL Proof systems

- Fitch (with DC,IND) is sound for RL
  - If $\Sigma \vdash_{Fitch} \varphi$ then $\Sigma \models_{RL} \varphi$
- Not complete for RL
  - Sometimes $\Sigma \models_{RL} \varphi$, but $\Sigma \nvdash_{Fitch} \varphi$
- Proves everything FOL proves, and more!


Emulation
FOL can be emulated with RL
Let L be a language with at least one object constant
Let L⁺ be L augmented by a new unary function n(.)
For Σ, φ over L,
Σ ⊨FOL φ if and only if Σ ⊨RL φ over L⁺
*FOL without equality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Relational Logic

- Simple semantic
- Precise in describing worlds
- Powerful proof system
- Can emulate FOL

- FOL
  - Logic of theoretical mathematics
- RL
  - Logic for common use