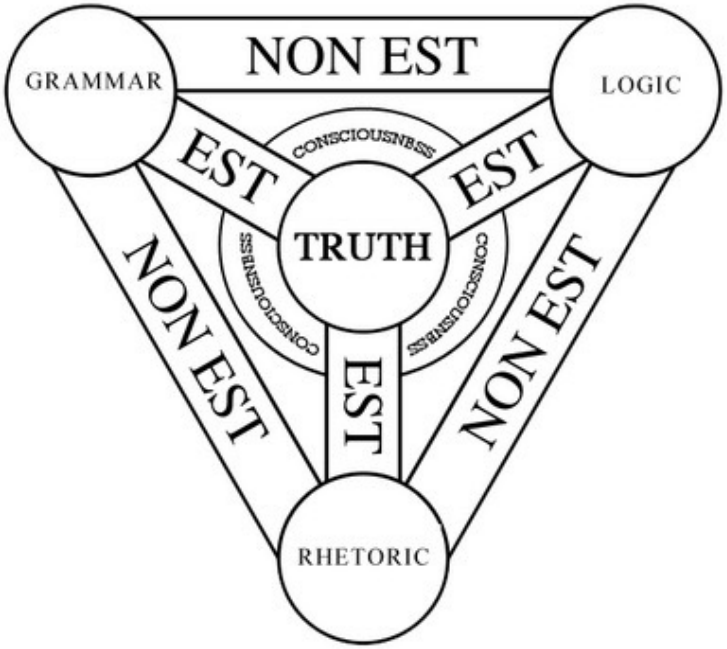
Introduction to Logic
Introduction

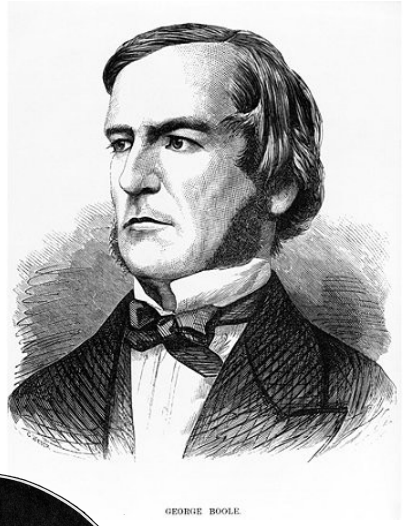
Michael Genesereth
Computer Science Department
Stanford University

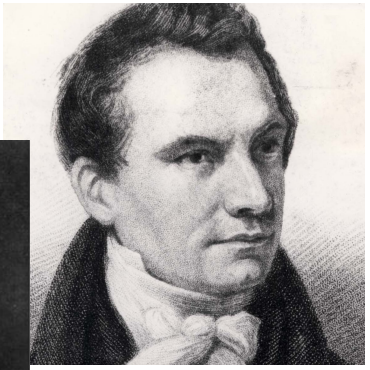
Lecture will begin at ~13:35 PDT.

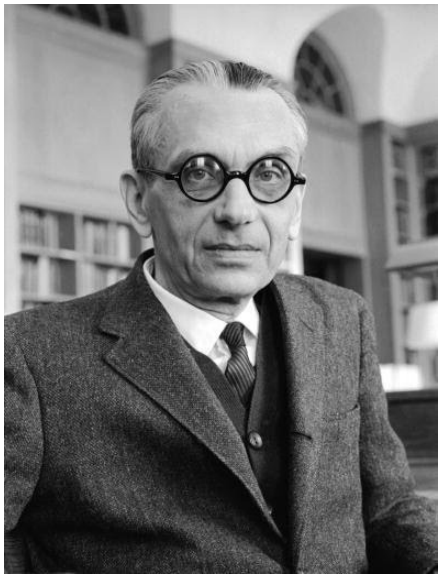
Greek Trivium

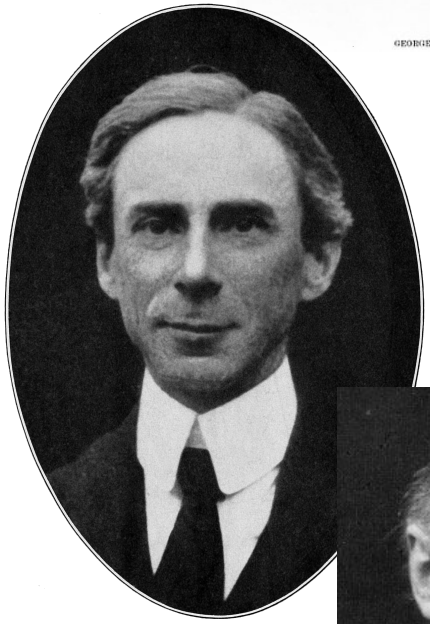


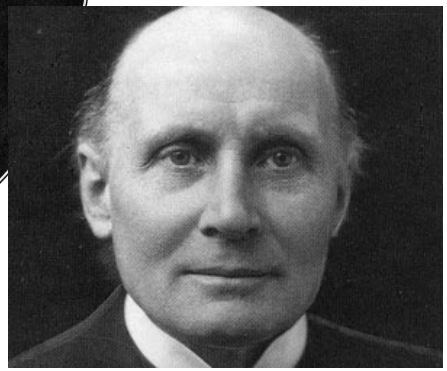
History



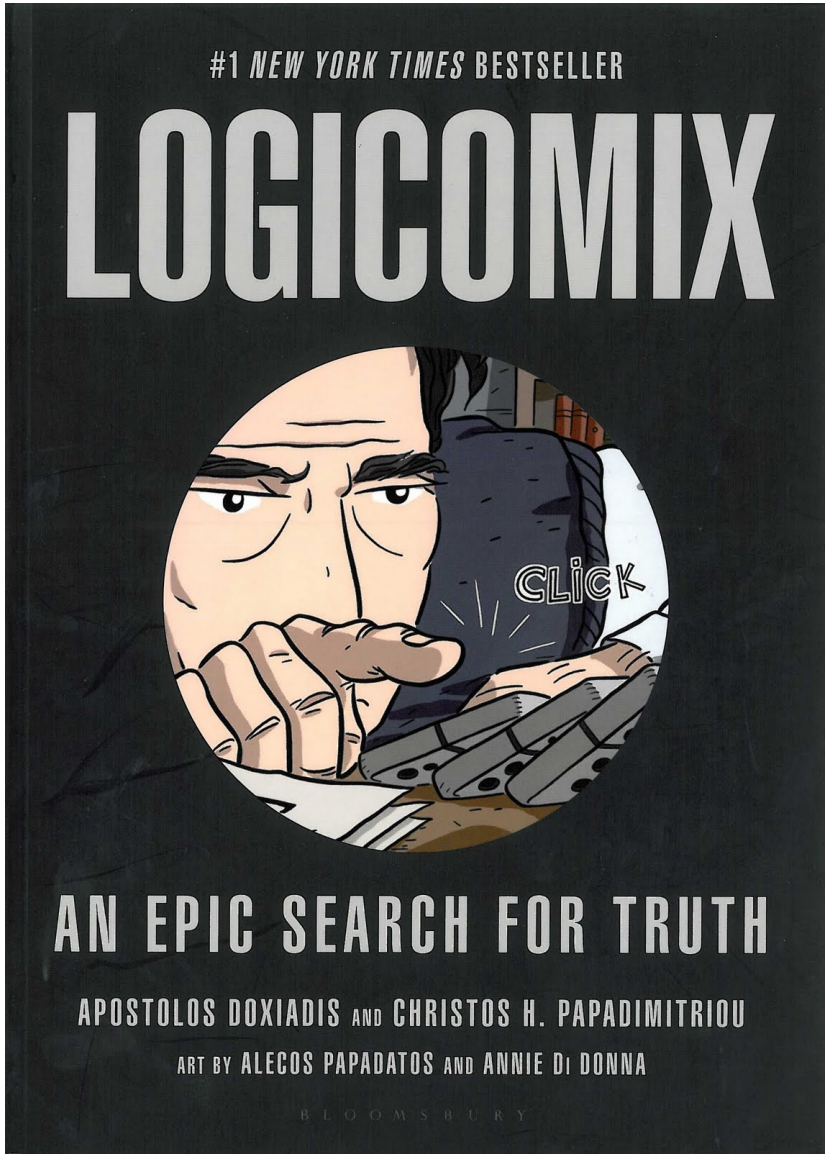








History of Logic



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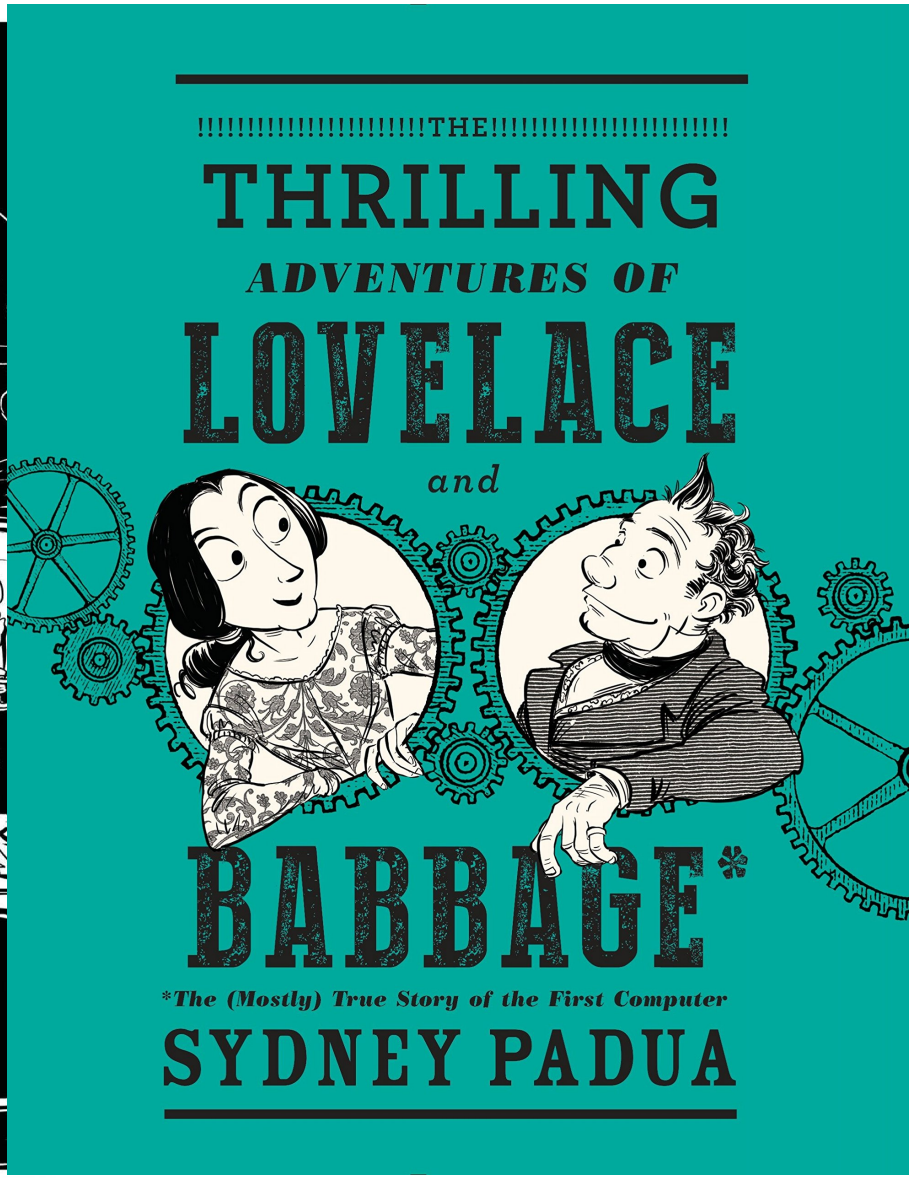


History of Logic

Uses of Logic

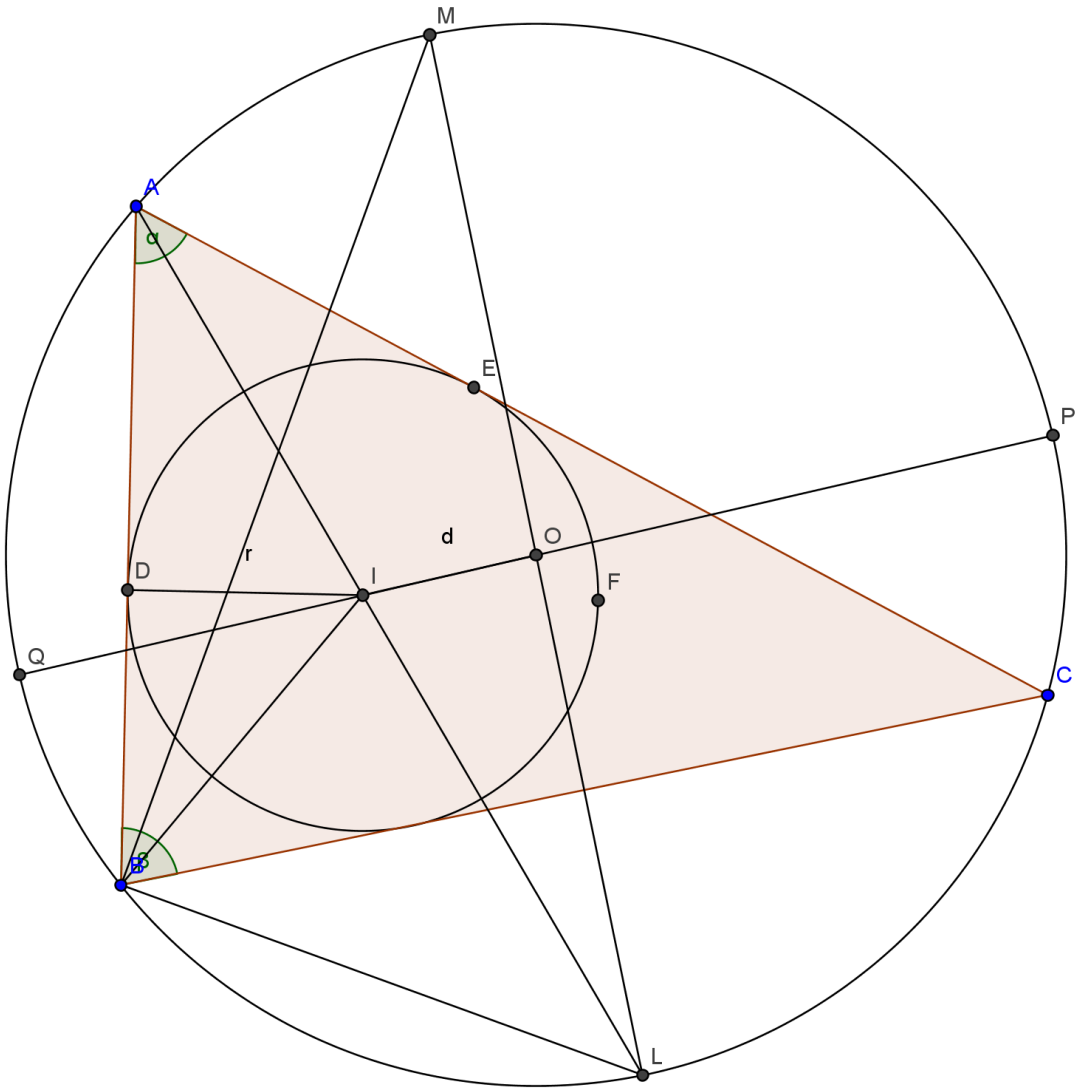
Mathematics

Euler's theorem:
The distance d between the circumcenter and the incenter in any triangle is given by $d^2 = R(R - 2r)$, where R is the circumradius, and r is the inradius.

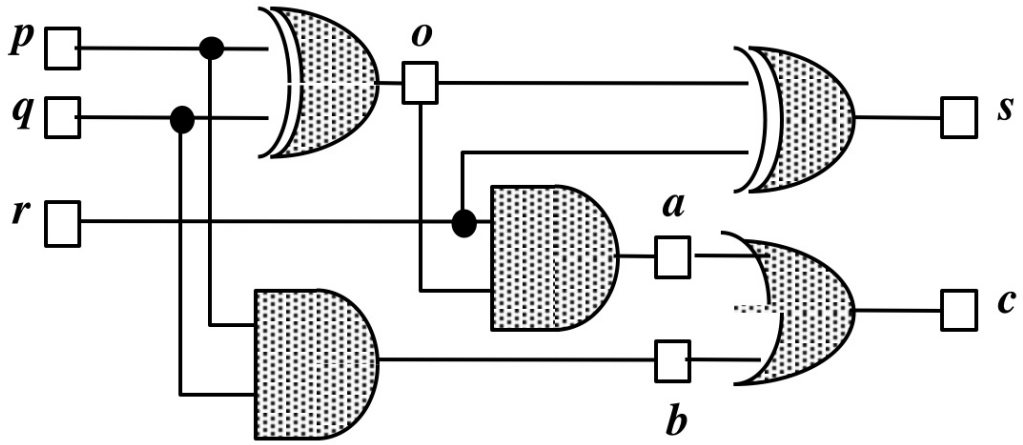
Proof:
Let O be the circumcentre of ΔABC, and I be its incentre, the extension of AI intersects the circumcircle at L, then L is the mid-point of arc BC (because AI intersects angle BAC).
Join LO and extend it so that it intersects the circumcircle at M.
From I construct a perpendicular to AB, and let D be its foot, then ID = r. It is not difficult to prove that $\Delta ADI \sim \Delta MBL$, so ID / BL = AI / ML, i.e. ID × ML = AI × BL.
Therefore
(1) 2Rr = AI × BL.
Join BI, because

angle BIL = α/2 +β/2,
angle IBL = β/2 + α/2,

therefore angle BIL = angle IBL, so BL = IL, and AI × IL = 2Rr (from (1)). Extend OI so that it intersects the circumcircle at P and Q, then PI × QI = AI × IL = 2Rr,
so (R + d)(R − d) = 2Rr, i.e. $d^2 = R(R - 2r)$.
Q.E.D

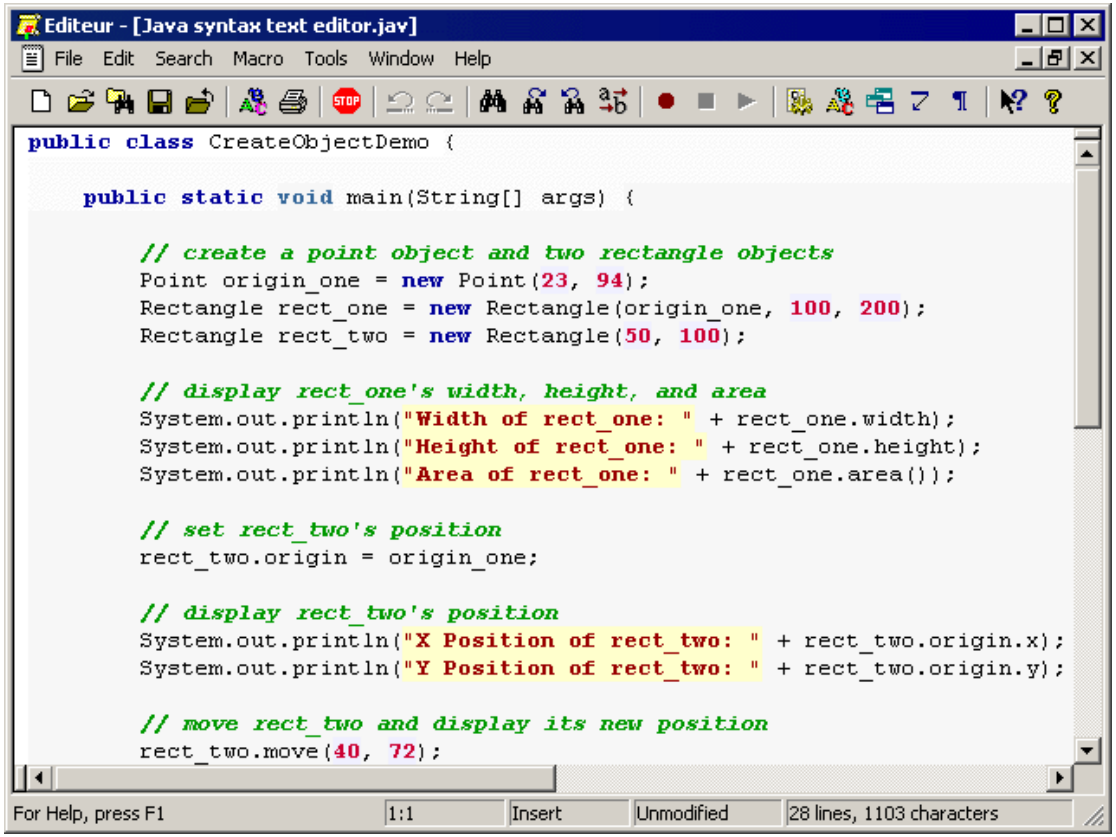


Hardware Engineering



What are the outputs for given inputs?
What inputs produce given outputs?
If its behavior is incorrect, which component is broken?
How many inputs do we need to confirm correct operation?

Software Engineering

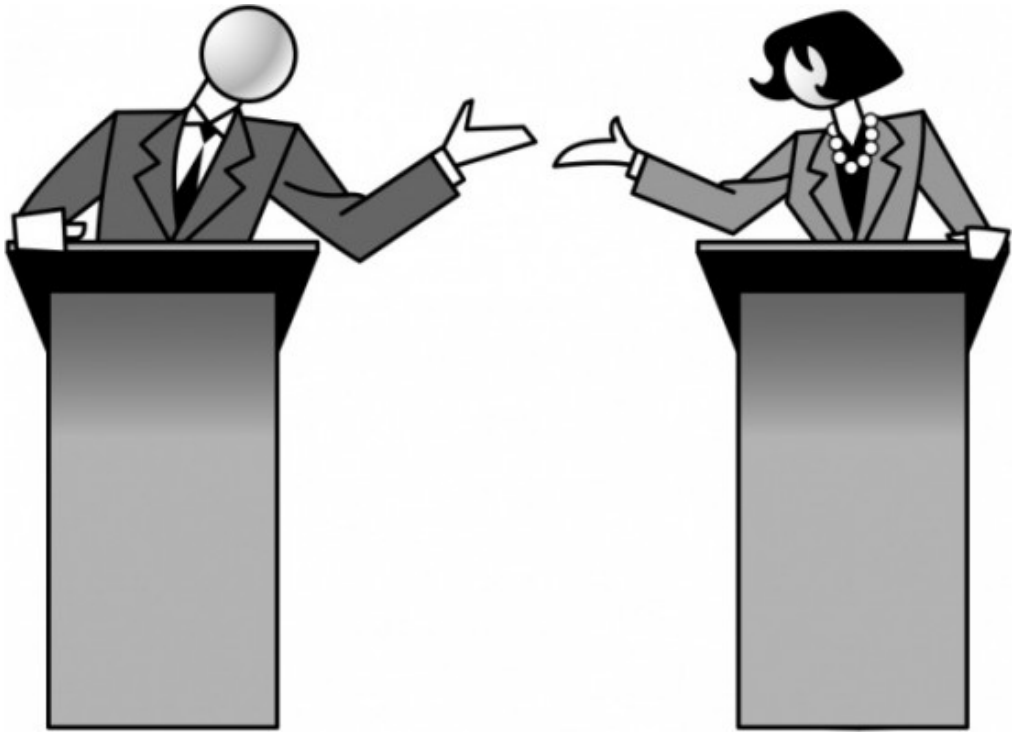


Partial Evaluation
Program verification
Debugging
Automatic Coding

Rules and Regulations

Games

Debate



Communication

Space-indexed Dynamic Programming: Learning to Follow Trajectories

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Adam Coates ACOATES@CS.STANFORD.EDU
Andrew Y. Ng ANG@CS.STANFORD.EDU
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Theorem 3.1 [following (Bagnell et al., 2004)] *Suppose* $\pi = (\pi_1, \ldots, \pi_D)$ *is a policy returned by an* ϵ*-approximate version of state-indexed PSDP where on each step the algorithm obtains* π_d *such that*

$$E_{s\sim\mu_d}[V_{\pi_d,\pi_{d+1},\ldots,\pi_D}(s)] \geq \arg\max_{\pi\in\Pi} E_{s\sim\mu_d}[V_{\pi,\pi_{d+1}\ldots\pi_D}(s)] - \epsilon$$

Then for all $\pi_{\text{ref}} \in \Pi^D$,

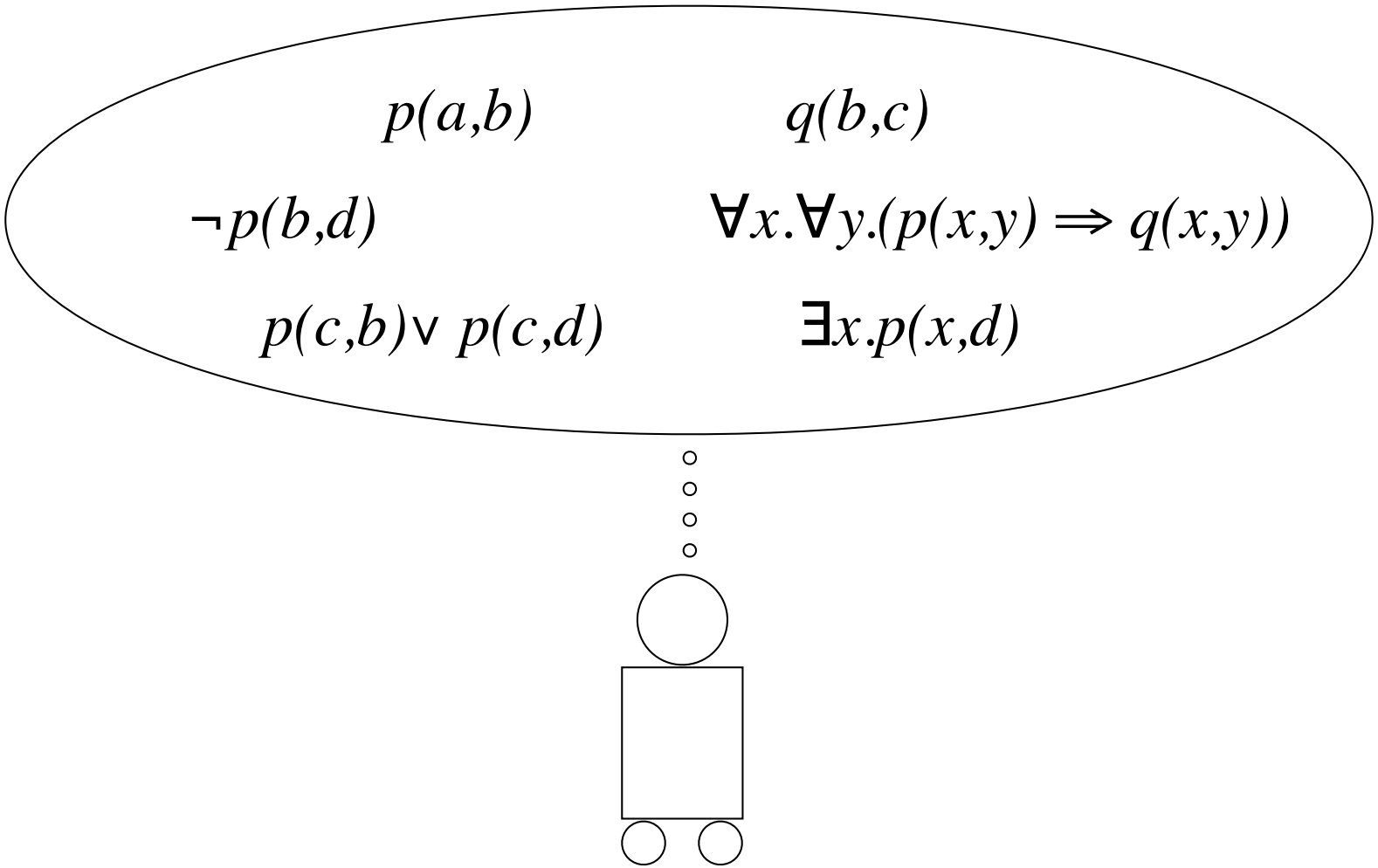
$$V_\pi(s_0) \geq V_{\pi_{\text{ref}}}(s_0) - D\epsilon - Dd_{\text{var}}(\mu, \mu_{\pi_{\text{ref}}})$$

where μ *is the baseline distribution over space-index states (without the time component) provided to SI-PSDP,* d_{var} *denotes the average variational distance, and* $\mu_{\pi_{\text{ref}}}$ *is the state distribution induced by* π_{ref}.

Logic and Computer Science

Logic is the mathematics of Computer Science
as
Calculus is the mathematics of Physics.

Computational Logic



Applications

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Proof:
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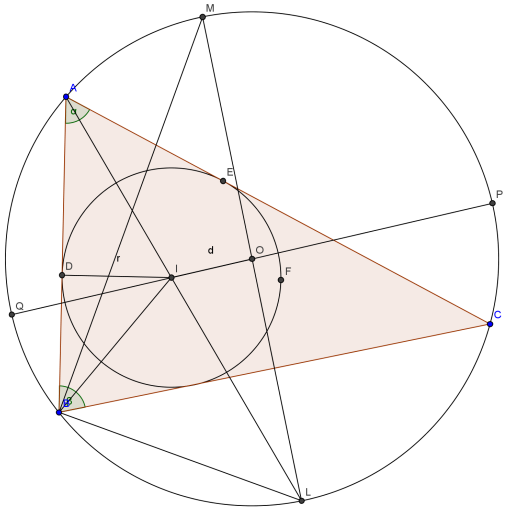
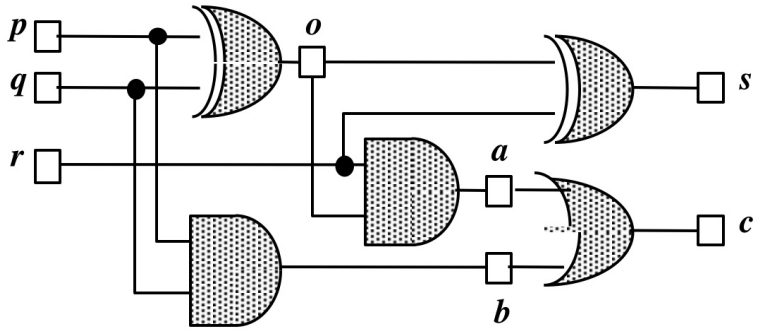
Q.E.D





Elements of Logic

Elements of Logic

Logical Language

Definitions: *A triangle is a polygon with three sides.*
Constraints: *Parents are older than their children.*
Partial Information: *Abby likes one of Cody or Dana.*

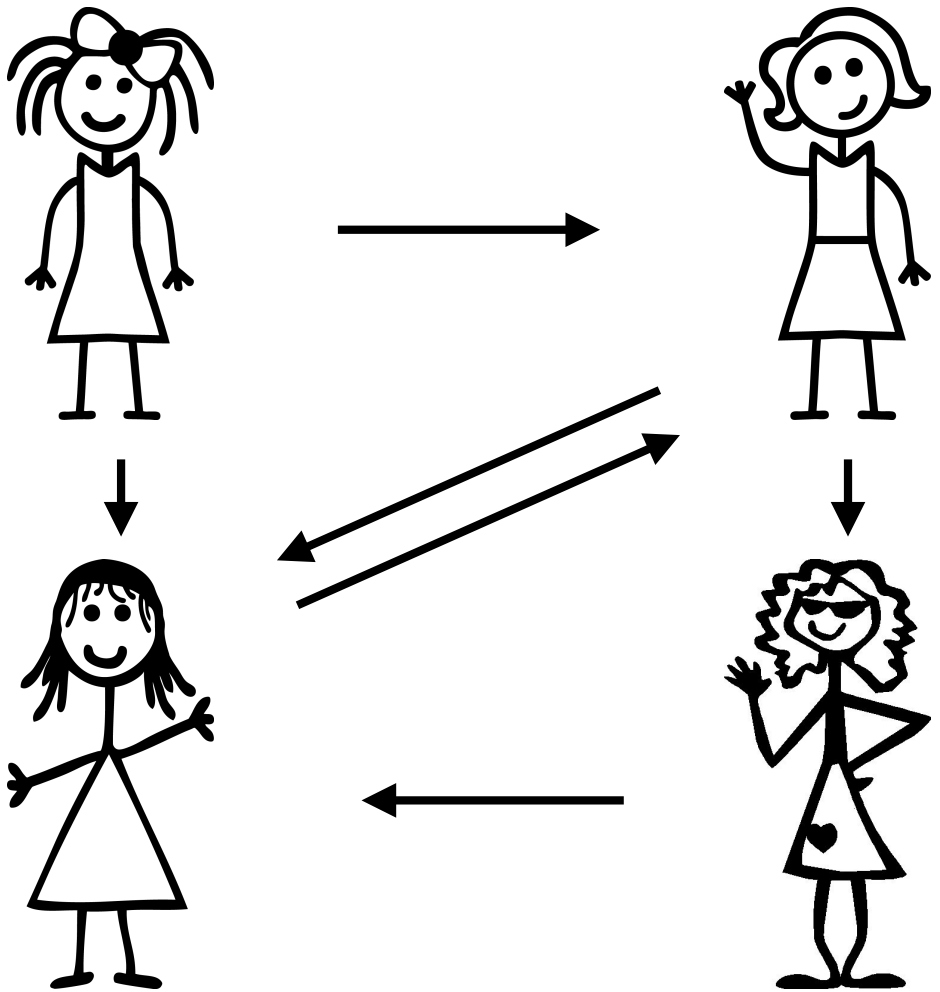
Logical Reasoning

Model Checking - truth tables, logic grids
Symbolic Manipulation - formula transformations, proofs

"Metalevel" Concepts and Analysis

Validity, Contingency, Unsatisfiability
Equivalence, Entailment, Consistency
Soundness, Completeness, Decidability

Friends



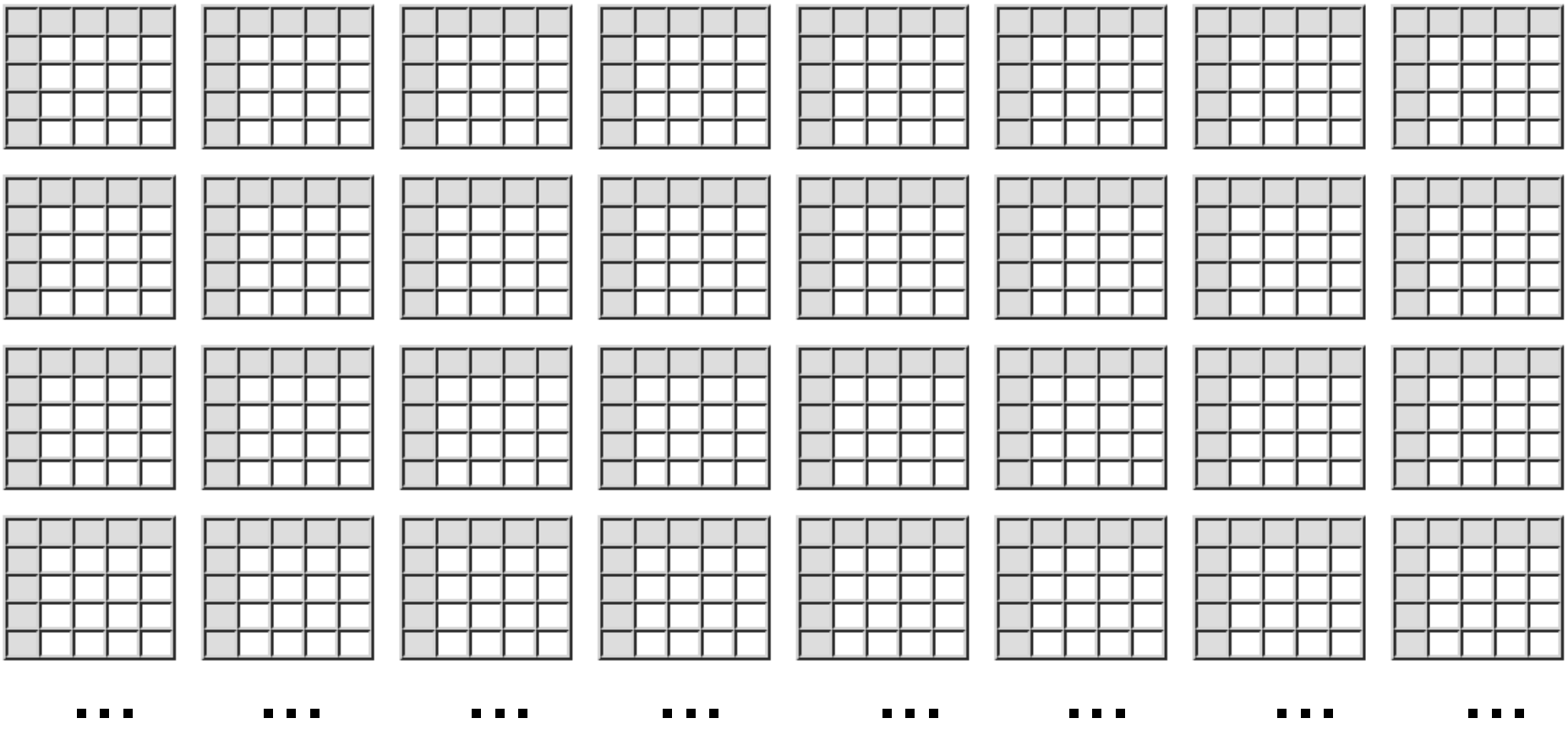
Friends

	Abby	Bess	Cody	Dana
Abby			✓	
Bess			✓	
Cody	✓	✓		✓
Dana			✓	

Friends

	Abby	Bess	Cody	Dana
Abby	✓		✓	
Bess		✓		✓
Cody	✓		✓	
Dana		✓		✓

Possible Worlds



Logical Language

Dana likes Cody.

*Abby does **not** like Dana.*

*Dana does **not** like Abby.*

*Bess likes Cody **or** Dana.*

*Abby likes **everyone** that Bess likes.*

*Cody likes **everyone** who likes her.*

***No one** likes herself.*

Logical Conclusions

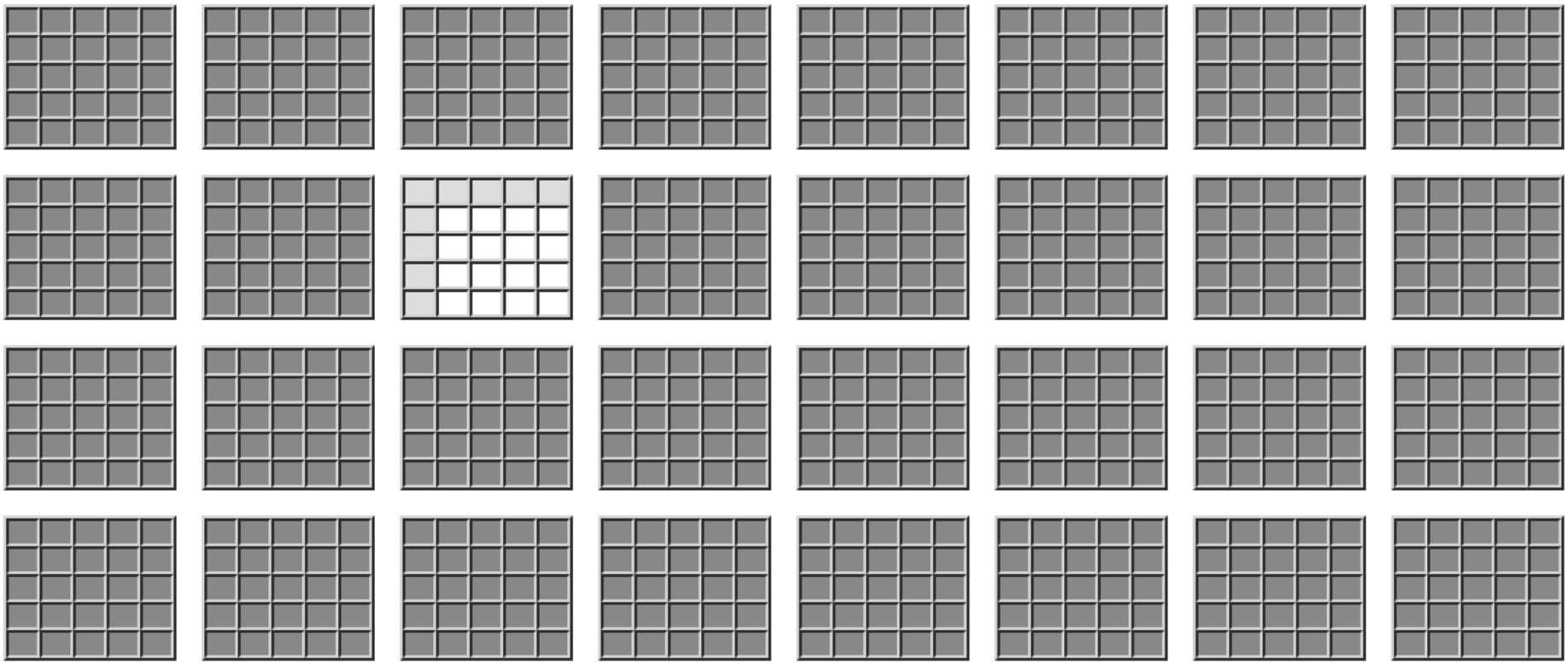
Premises:

Dana likes Cody.
Abby does not like Dana.
Dana does not like Abby.
Bess likes Cody or Dana.
Abby likes everyone that Bess likes.
Cody likes everyone who likes her.
No one likes herself.

Questions:

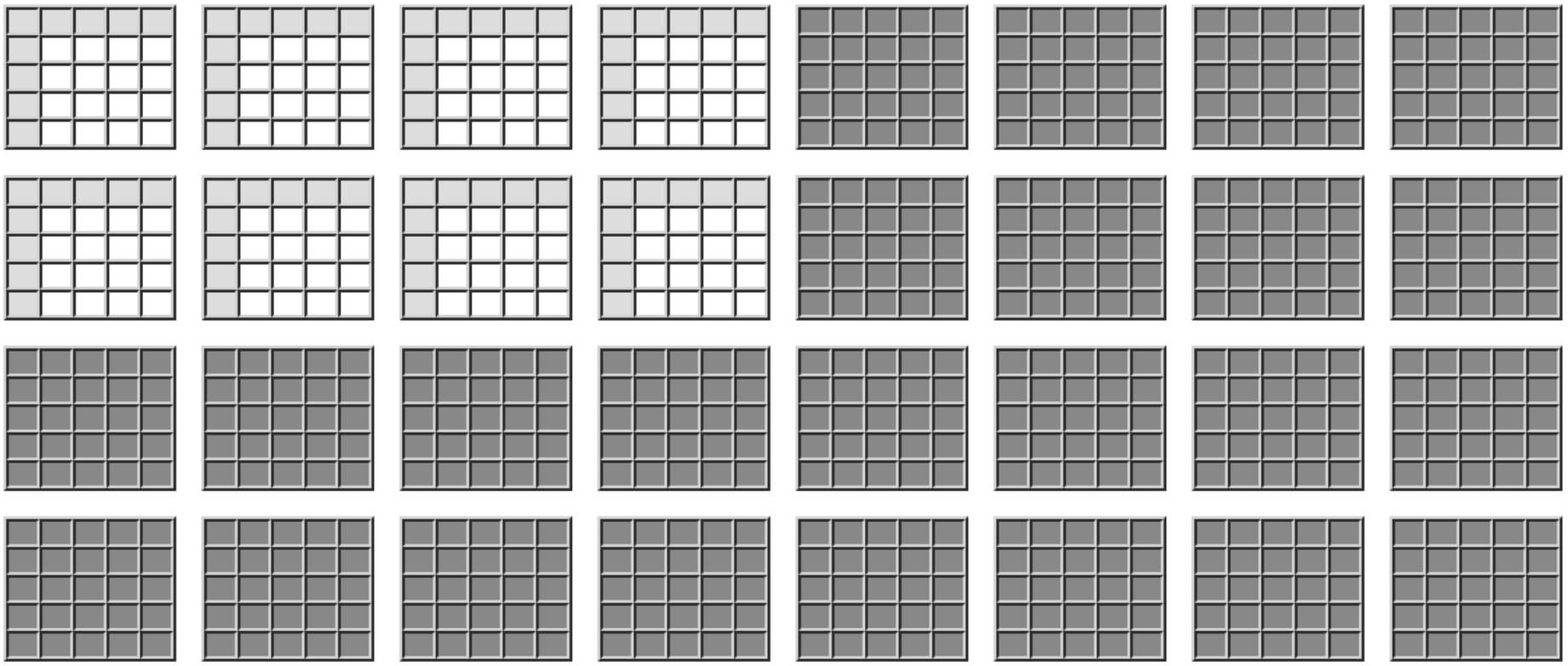
Does Bess likes Cody? *Yes*
Does Bess like Dana? *No*
Does Dana like Bess? *Maybe*
Does everybody likes someone? *Yes*
Does someone like everyone? *No*

One Specific World



Special case with ***complete information*****.**

Multiple Possible Worlds



In Logic, we typically need to deal with ***incomplete information****.*

Logical Entailment

A set of premises *logically entails* a conclusion if and only if *every* world that satisfies the premises satisfies the conclusion.

Logical Conclusions

Givens:

Dana likes Cody.
Abby does not like Dana.
Dana does not like Abby.
Bess likes Cody or Dana.
Abby likes everyone that Bess likes.
Cody likes everyone who likes her.
No one likes herself.

Questions:

Does Bess likes Cody? *Yes*
Does Bess like Dana? *No*
Does Dana like Bess? *Maybe*
Does everybody likes someone? *Yes*
Does someone like everyone? *No*

	Abby	Bess	Cody	Dana
Abby			✓	
Bess			✓	
Cody	✓	✓		✓
Dana			✓	

	Abby	Bess	Cody	Dana
Abby		✓	✓	
Bess			✓	
Cody	✓	✓		✓
Dana			✓	

	Abby	Bess	Cody	Dana
Abby			✓	
Bess			✓	
Cody	✓	✓		✓
Dana		✓	✓	

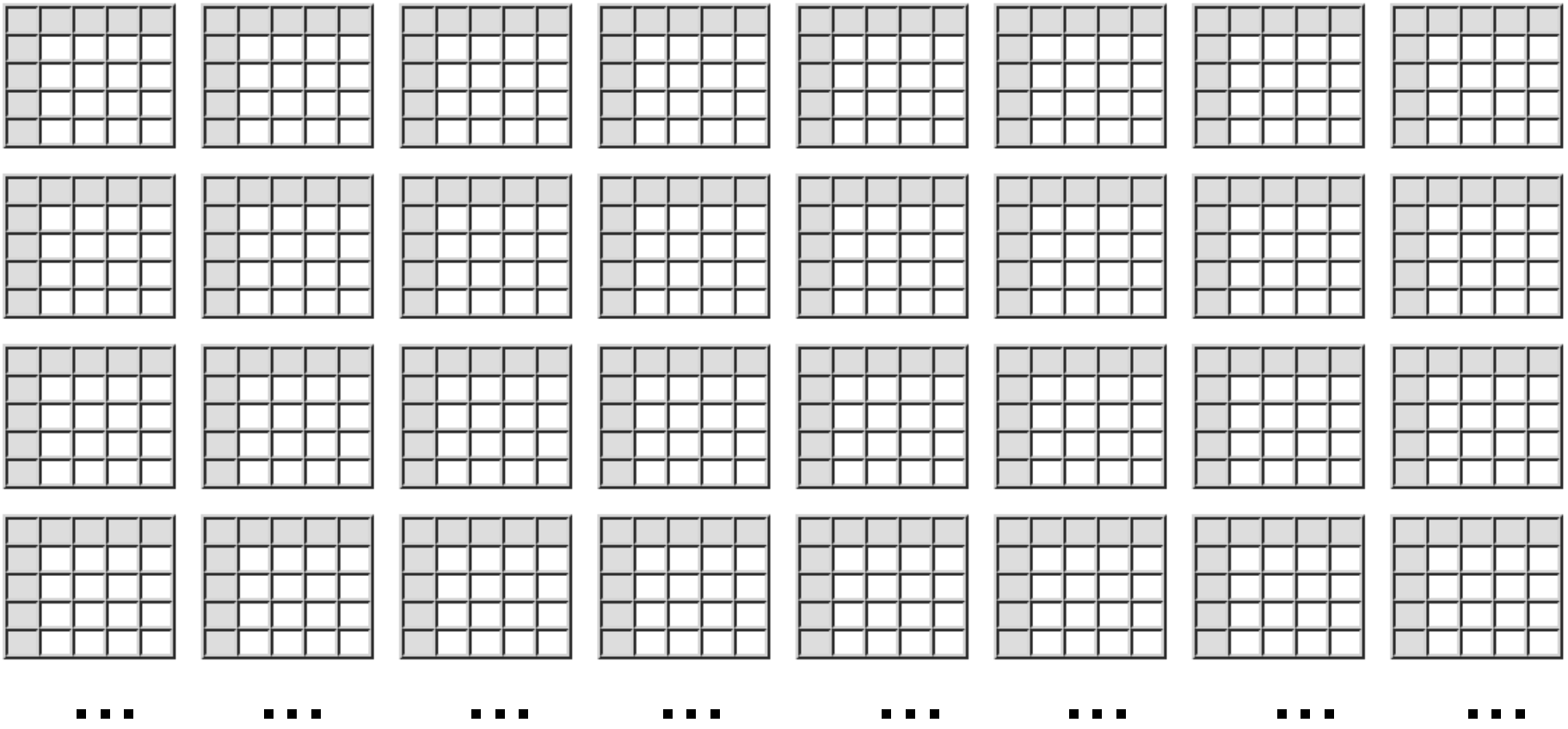
	Abby	Bess	Cody	Dana
Abby		✓	✓	
Bess			✓	
Cody	✓	✓		✓
Dana		✓	✓	

Model Checking

Iterate through *all* possible worlds. For every world that satisfies the premises, check if it satisfies the conclusion.

Problem with Model Checking

Lots of Worlds (sometimes infinitely many)



Model Checking is like solving polynomial equations by enumerating all possible values for the variables.

Symbolic Manipulation

A *proof* is a sequence of sentences in which every sentence is either a premise or the result of applying a *rule of inference* to earlier elements of the sequence.

Rules of Inference

A *rule of inference* is a reasoning pattern consisting of some premises and some conclusions.

In other words, if we believe the premises, a rule of inference tells us that we should also believe the conclusions.

Symbolic manipulation rather than model checking.

Sample Rule of Inference

All of Abby's friends are Bess's friends.
All of Bess's friends are Cody's friends.
***Therefore**, all of Abby's friends are Cody's friends.*

Sample Rule of Inference

All Accords are Hondas.
All Hondas are Japanese.
***Therefore**, all Accords are Japanese.*

Sample Rule of Inference

All borogoves are slithy toves.
All slithy toves are mimsy.
***Therefore**, all borogoves are mimsy.*

General Rule of Inference

All *x are y.*
All *y are z.*
Therefore, ***all*** *x are z.*

Bertrand Russell

Logic "may be defined as the subject in which we never know what we are talking about nor whether what we are saying is true."

- Bertrand Russell

Unsound Rule of Inference

All x are y.
Some y are z.
***Therefore**, some x are z.*

No! No!! No!!!

Using Unsound Rule of Inference

All Toyotas are Japanese cars.
Some Japanese cars are made in America.
***Therefore**, some Toyotas are made in America.*

Sometimes produces a result that *happens* to be true.

Using Unsound Rule of Inference

All Toyotas are cars.
Some cars are Porsches.
***Therefore**, some Toyotas are Porsches.*

Sometimes produces a result that *happens* to be false.

Provability

A set of premises *logically entails* a conclusion if and only if every world that satisfies the premises satisfies the conclusion.

A conclusion is *provable* from a set of premises if and only if there is a finite sequence of sentences in which every element is either a premise or the result of applying a *sound* rule of inference to earlier members in the sequence.

Soundness and Completeness

As we shall see, for well-behaved logics, logical entailment and provability are identical - a set of premises **logically entails** a conclusion *if and only if* the conclusion is **provable** from the premises.

This is a very big deal.

Deduction

A rule of inference is *sound* if and only if the conclusion is true whenever the premises are true.

The application of sound rules of inference is called *deduction*.

Induction

Induction is reasoning from the specific to the general.

> *I have seen 1000 black ravens.*
> *I have never seen a raven that is not black.*
> *Therefore, every raven is black.*

Induction is not necessarily sound (but it can be useful).

Induction versus Deduction

Induction is the basis for **Science** (and machine learning)
Deduction is the subject matter of **Logic**.

Science aspires to discover / propose **new** knowledge.
Logic aspires to apply and/or analyze **existing** knowledge.

Niels Bohr to Albert Einstein

"You are not thinking; you are just being logical."

Symbolic Logic

Logical Sentences

Dana likes Cody.

*Abby does **not** like Dana.*

*Dana does **not** like Abby.*

*Bess likes Cody **or** Dana.*

*Abby likes **everyone** that Bess likes.*

*Cody likes **everyone** who likes her.*

***Everyone** likes herself.*

Complexity of Natural Language

One grammatically correct sentence:

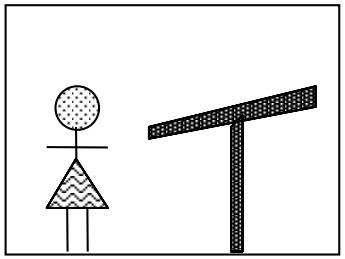
The cherry blossoms in the spring.

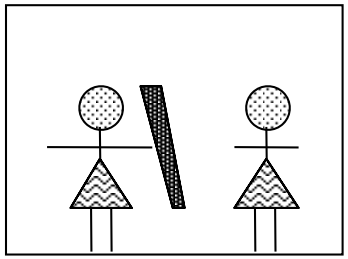
Another grammatically correct sentence:

The cherry blossoms in the spring sank.

Grammatical Ambiguity

There's a girl in the room with a telescope.





Newseum Headlines

Crowds Rushing to See Pope Trample 6 to Death

Newseum Headlines

Crowds Rushing to See Pope Trample 6 to Death

Scientists Grow Frog Eyes and Ears

Newseum Headlines

Crowds Rushing to See Pope Trample 6 to Death

Scientists Grow Frog Eyes and Ears

Fried Chicken Cooked in Microwave Wins Trip

Newseum Headlines

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Scientists Grow Frog Eyes and Ears

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Indian Ocean Talks

Misteaks in Print

Residents report that a hole was cut in the fence surrounding a nudist colony. Police are ***looking into*** *it.*

Doug Lenat's Logic

Champagne is better than beer.
Beer is better than soda.
***Therefore**, champagne is better than soda.*

X is better than Y.
Y is better than Z.
***Therefore**, X is better than Z.*

Bad sex is better than nothing.
Nothing is better than good sex.
***Therefore**, bad sex is better than good sex.*
Really?

Logistics

Logical Extensions

Language

Probabilities

Metaknowledge - knowledge about knowledge

Paradoxes, e.g. *This sentence is false*.

Reasoning

Negation as Failure - *knowing not* versus *not knowing*

Induction, Abduction, Analogical Reasoning

Paraconsistent Reasoning - reasoning with inconsistency

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Logical Reasoning

Model Checking - truth tables, logic grids
Symbolic Manipulation - formula transformations, proofs

"Metalevel" Concepts and Analysis

Validity, Contingency, Unsatisfiability
Equivalence, Entailment, Consistency
Soundness, Completeness, Decidability

Multiple Logics

Propositional Logic (logical operators)

If *it is raining* ***and*** *it is cold,* ***then*** *the ground is wet.*

Relational Logic (variables and quantifiers)

If $\boldsymbol{x}$ *is a parent of* $\boldsymbol{y}$*, then* $\boldsymbol{x}$ *is older than* $\boldsymbol{y}$*.*

Functional Logic (compound terms)

$\{a, b\}$ *is a subset of* $\{a, b, c\}$*.*

Schedule

Week	Tuesday	Thursday
1	September 26 Introduction	September 28 Propositional Logic
2	October 3 Propositional Analysis	October 5 Direct Proofs
3	October 10 Natural Deduction	October 12 Refutation Proofs
4	October 17 Review	October 19 Quiz 1
5	October 24 Relational Logic	October 26 Relational Analysis
6	October 31 Fitch Proofs	November 2 Review
7	November 7 No Class	November 9 Quiz 2
8	November 14 Functional Logic	November 16 Induction
	Thanksgiving Week	
9	November 28 Equality	November 30 Review
10	December 6 No Class	December 8 Quiz 3
11	December 12 3:30-6:30 Optional Final	

Grades

Numerical Grade

40% - quiz on Propositional Logic (Week 4)
30% - quiz on Relational Logic (Week 7)
30% - quiz on Functional Logic (Week 10)

NB: We will LIKELY offer an optional final exam for those wishing to improve their quiz grades.

Letter Grade

Based on numerical grade (see above)
No curve - i.e. independent of number of students
A, B, C distributed uniformly over 70% - 100%

***Discretionary* Extra Credit**

class attendance, Ed forum, puzzles, ...

Course Website

http://cs157.stanford.edu

http://intrologic.stanford.edu/stanford

Hints on How to Take the Course

Materials of the Course

- Lectures
- Textbook / Lessons
- Exercises
- Puzzles
- Tools

Read the notes.
Do the exercises.
Do the exercises!
Do the exercises!!
*Learn **actively**.*

Ed Discussion

- Read discussion
- Post questions
- Answer questions

Working in groups is okay / recommended!!

Biggest Mistake

2022 Quiz 1 Mean Score

80.8

Secret Word

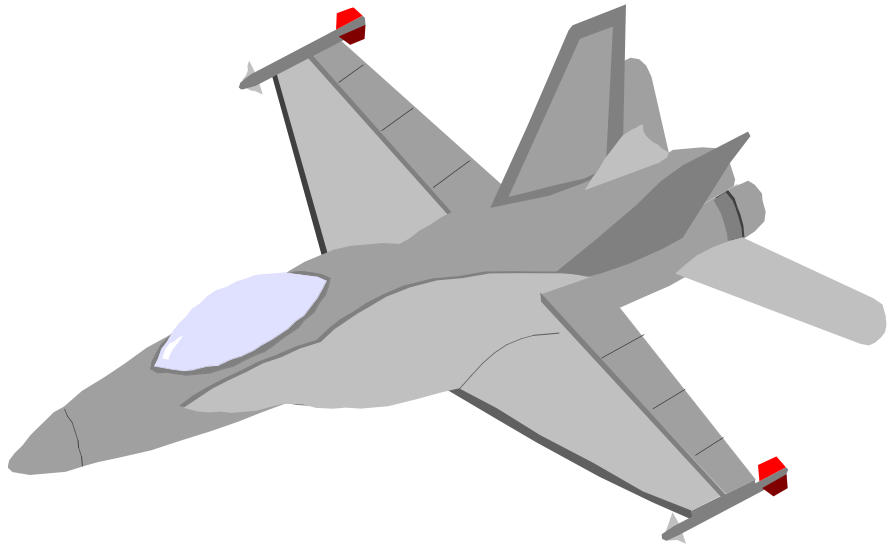
"Fallacy"

Secret Word

spoken, not written

Value of Practice

Value of Theory



Mike took it twice!