

# Logica

*Tools  
for  
Logic*

Prove  $\sim p \mid \sim q$  from the premise  $\sim(p \ \& \ q)$

Goal:  $\sim p \mid \sim q$

**Intuition:** Okay so the goal is a disjunction. One possibility is to prove using OR Elimination.

Another possibility: to use Negation Introduction + Negation Elimination to prove  $\sim p \mid \sim q$ . Observe that in Fitch-Example2 you already proved  $(p \ \& \ q)$  from the premise  $\sim p \mid \sim q$ . Therefore let's proceed with Negation Introduction + Negation Elimination.

High-level Approach:

1. Assume  $\sim(\sim p \mid \sim q)$
2. Establish a contradiction i.e.
  - Prove  $\sim(\sim p \mid \sim q) \Rightarrow \sim(p \ \& \ q)$
  - Prove  $\sim(\sim p \mid \sim q) \Rightarrow (p \ \& \ q)$
3. Use **Negation Introduction** to prove  $\sim\sim(\sim p \mid \sim q)$  and then **Negation Elimination** to prove  $\sim p \mid \sim q$

**Note** that in Step 5, we have taken a shortcut, since we have already shown how the results can be proved in **Fitch-Example2**. Note that in the problems you **CANNOT USE SHORTCUTS** - you actually have to plug in the proof of  $\sim(\sim p \mid \sim q) \Rightarrow (p \ \& \ q)$

Show Instructions

## Fitch

- |                             |   |                             |
|-----------------------------|---|-----------------------------|
| <input type="checkbox"/> 1. | $\sim(p \ \& \ q)$                                      | Premise                     |
| <input type="checkbox"/> 2. | $\sim(\sim p \mid \sim q)$                              | Assumption                  |
| <input type="checkbox"/> 3. | $\sim(p \ \& \ q)$                                      | Reiteration: 1              |
| <input type="checkbox"/> 4. | $\sim(\sim p \mid \sim q) \Rightarrow \sim(p \ \& \ q)$ | Implication Introduction: 3 |
| <input type="checkbox"/> 5. | $\sim(\sim p \mid \sim q) \Rightarrow p \ \& \ q$       | Proved in Fitch-Example2    |
| <input type="checkbox"/> 6. | $\sim\sim(\sim p \mid \sim q)$                          | Negation Introduction: 5, 4 |
| <input type="checkbox"/> 7. | $\sim p \mid \sim q$                                    | Negation Elimination: 6     |

Premise	Negation Introduction	Implication Introduction	Universal Introduction
Assumption	Negation Elimination	Implication Elimination	Universal Elimination
Reiteration	And Introduction	Biconditional Introduction	Existential Introduction
Shortcut	And Elimination	Biconditional Elimination	Existential Elimination
Delete	Or Introduction	Equality Introduction	
	Or Elimination	Equality Elimination	

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