

Fitch

Prove q from the sentences $p \vee q$ and $\sim p$

Goal: q

Intuition: From the premises, it is clear that p is not true. Therefore, q must be true. Now how to prove it? We can perhaps prove it on a *case by case* basis.

The premise $p \vee q$ says that at least one of p or q is true.

Case 1: If p is true, then we prove that q is true.

Case 2: If q is true, then we're done. This *case by case* proof is exactly what **OR Elimination** is.

High-level Approach:

1. Prove $p \Rightarrow q$
2. Prove $q \Rightarrow q$
3. Use **OR Elimination** (with premise $p \vee q$)

Proving $p \Rightarrow q$ [Steps 3-12]

- **Assume** p and prove q .

Observation: By assuming p , we are introducing a contradiction since $\sim p$ is a premise.

1.	$p \vee q$	Premise
2.	$\sim p$	Premise
3.	p	Assumption
4.	$\sim q$	Assumption
5.	$\sim p$	Reiteration: 2
6.	$\sim q \Rightarrow \sim p$	Implication Introduction: 5
7.	$\sim q$	Assumption
8.	p	Reiteration: 3
9.	$\sim q \Rightarrow p$	Implication Introduction: 8
10.	$\sim \sim q$	Negation Introduction: 9, 6
11.	q	Negation Elimination: 10
12.	$p \Rightarrow q$	Implication Introduction: 11
13.	q	Assumption
14.	$q \Rightarrow q$	Implication Introduction: 13
15.	q	Or Elimination: 1, 12, 14

Premise	Negation Introduction	Implication Introduction	Universal Introduction
Assumption	Negation Elimination	Implication Elimination	Universal Elimination
Reiteration	And Introduction	Biconditional Introduction	Existential Introduction