## General Game Playing Incomplete Search

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## Game Variety

## Small Games



Large Games


## Complete Game Graph Search

## Incomplete Search



## Evaluation of Non-Terminal States



How do we evaluate non-terminal states?

## Choice of Depth



To what depth should we search?

## Variable Depth Search



Should we search different branches to different depths?

## Persistence



Can we preserve results across moves?

## Evaluation Functions

How do we evaluate non-terminal states?

## Evaluation Functions

Chess examples:
Piece count
Board control

Comments
Not necessarily successful
Game-specific but this is general game playing

## Heuristic \#1 - Mobility / Focus

Mobility is a measure of the number of things a player can do. Focus is a measure of the narrowness of the search space. It is the opposite of mobility.

Basis - number of actions in a state or number of states reachable from that state. Horizon - current state or $n$ moves away.

Sometimes it is good to focus to cut down on search space. Often better to restrict opponents' moves while keeping one's own options open.

## Heuristic \#1 - Mobility / Focus

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## Implementation

```
function mobility (state)
    {var actions = findlegals(state,library);
        var feasibles = findactions(library);
    return (actions.length/feasibles.length * 100)}
function focus (state)
    {var actions = findlegals(state,library);
    var feasibles = findactions(library);
    return (100 - actions.length/feasibles.length * 100)}
```


## GGP-06 Final - Cylinder Checkers



## Heuristic \#2 - Pessimism

Assume value of 0 for non-terminal states.

$$
\begin{array}{ll}
\operatorname{value}(\text { state })=\operatorname{goal}(\text { role }, \text { state }) & \\
\text { if terminal(state }) \\
\text { value }(\text { state })=0 & \\
\text { otherwise }
\end{array}
$$

## Example



Grey - estimates of rewards in non-terminal states - here 0. Black - rewards in terminal states.

## Heuristic \#3 - Intermediate Values

Assume reward for non-terminal states.

$$
\text { value }(\text { state })=\text { goal(role,state })
$$

Good on monotonic games (where utility accumulates as the game progresses), e.g. alquerque.

Not so good on nonmonotonic games. Susceptible to "false summits".

## Example



Blue - rewards in non-terminal states. Black - rewards in terminal states.

## Weighted Linear Combinations

Definition

$$
f(s)=w_{1} \times f_{1}(s)+\ldots+w_{n} \times f_{n}(s)
$$

Examples:
Final State Value when known
Mobility / Focus
Intermediate State Values
Other

Some players estimate weights by experimentation during the start clock. More on this in a few weeks.

## Weighted Linear Combinations

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Examples:
Mobility / Focus
Intermediate State Values
Other

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## Depth-Limited Search

To what depth should we search?

## Depth-Limited Search

## Depth



## Depth-Limited Minimax

## Minimax:

```
function minimax (state)
    {if (findterminalp(state,library))
    {return findreward(role,state,library)*1};
    var active = findcontrol(state,library);
    if (active===role) {return maximize(state)};
    return minimize(state)}
```


## Depth-Limited Minimax

```
function minimaxdepth (state,depth)
    {if (findterminalp(state,library))
        {return findreward(role,state,library)*1};
    if (depth<=O) {return evalfun(state,library)};
    var active = findcontrol(state,library);
    if (active===role) {return maxscore(state,depth-1)};
    return minscore(state,depth-1)}
```


## maxscore and minscore

```
function maxscore (state,depth)
    {var actions = findlegals(state,library);
    if (actions.length===0) {return 0};
    var score = 0;
    for (var i=0; i<actions.length; i++)
            {var newstate = simulate(actions[i],state,library);
            var newscore = minimaxdepth(newstate,depth);
            if (newscore===100) {return 100};
            if (newscore>score) {score = newscore}};
    return score}
function minscore (state,depth)
{var actions = findlegals(state,library);
    if (actions.length===0) {return 0};
    var score = 100;
    for (var i=0; i<actions.length; i++)
            {var newstate = simulate(actions[i],state,library);
            var newscore = minimaxdepth(role,newstate,depth);
            if (newscore===0) {return 0};
            if (newscore<score) {score = newscore}};
return score}
```


## Remarks

Legal and random players are degenerate depth-limited search with depth 0 .

Onestep and Twostep are degenerate depth-limited search with depths 1 and 2.

In general, we would like to allow greater depths.

## Problem



To what depth should we search?

## Problem - Insufficient Depth



## Problem - Excessive Depth



# Iterative Deepening 

To what depth should we search?

## Iterative Deepening

Use depth-limited search to explore entire tree to level 1 Use depth-limited search to explore entire tree to level 2 Use depth-limited search to explore entire tree to level 3 And so forth

Continue till time runs out
Choose action that gives maximal value

Level 1

Level 2


Level 3


## Naive Implementation

```
function playminimaxid ()
    {var best = findlegalx(state,library);
    for (var depth=1; depth<10; depth++)
            {var action = minimaxdepth(state,depth);
            best = action};
        return best}
```

At what depth do we stop?

## Implementation

```
function playminimaxid ()
    {var deadline = Date.now()+(playclock-1)*1000;
        var best = findlegalx(state,library);
        for (var depth=1; depth<10; depth++)
            {var action = playminimaxidinner(state,depth,deadline);
            if (action===false) {return best};
            best = action};
        return best}
```


## Implementation

```
function playminimaxidinner (state,depth,deadline)
    {var actions = shuffle(findlegals(state,library));
    var best = actions[0];
    var score = 0;
    for (var i=0; i<actions.length; i++)
    {var newstate = simulate(actions[i],state,library);
    var newscore = minimaxid(newstate,depth,deadline);
    if (newscore===false) {return false};
    if (newscore===100) {return actions[i]};
    if (newscore>score) {best = actions[i]; score=newscore}};
    return best}
```


## Implementation

```
function minimaxid (state,depth,deadline)
    {if (findterminalp(state,library))
    {return findreward(role,state,library)*1};
    if (depth<=0) {return evalfun(state,library)*1};
    if (Date.now()>deadline) {return false};
    if (findcontrol(state,library)===role)
    {return maxscoreid(state,depth,deadline)};
return minscoreid(state,depth,deadline)}
```


## maxscore and minscore

```
function maxscore (state,depth,deadline)
    {var actions = findlegals(state,library);
        if (actions.length===0) {return 0};
        var score = 0;
        for (var i=0; i<actions.length; i++)
            {var newstate = simulate(actions[i],state,library);
                var newscore = minimaxid(newstate,depth,deadline);
                if (newscore===false) {return false};
                if (newscore===100) {return 100};
                if (newscore>score) {score = newscore}};
    return score}
function minscore (state,depth,deadline)
    {var actions = findlegals(state,library);
        if (actions.length===0) {return 0};
        var score = 100;
        for (var i=0; i<actions.length; i++)
        {var newstate = simulate(actions[i],state,library);
        var newscore = minimaxid(newstate,depth,deadline);
        if (newscore===false) {return false};
        if (newscore===0) {return 0};
        if (newscore<score) {score = newscore}};
    return score}
```


## Advantages and Disadvantages

Advantages
requires storage linear in depth
still finds shortest path to an optimal solution
Disadvantages (?)
Repeated work

## but

Cost only a constant factor more than depth-first search
Why? Tree is growing exponentially, so fringe of tree and size of tree above fringe are approximately same

## More Information

https://en.wikipedia.org/wiki/
Iterative deepening depth-first search

## Monte Carlo Search

## Basic Idea

Sample a few branches of the game tree and use results to estimate values.
(1) Optionally explore game graph to some level.
(2) Beyond this, explore to end of game from fringe nodes, making random choices for moves of all players.
(3) Assign expected utilities to fringe states by summing utilities and dividing by number of trials.

Example


## Example



## Example



## mcs

```
function mcs (state,level)
    {if (findterminalp(state,library))
        {return findreward(role,state,library)*1};
    if (level>levels) {return montecarlo(state)};
    var active = findcontrol(state,library);
    if (active===role) {return maxscore(state,level+1)};
    return minscore(state,level+1)}
```


## maxscore and minscore

```
function maxscore (state,level)
    {var actions = findlegals(state,library);
    if (actions.length===0) {return 0};
    var score = 0;
    for (var i=0; i<actions.length; i++)
            {var newstate = simulate(actions[i],state,library);
            var newscore = mcs(newstate,level);
        if (newscore===100) {return 100};
        if (newscore>score) {score = newscore}};
    return score}
function minscore (state,level)
{var actions = findlegals(state,library);
    if (actions.length===0) {return 0};
    var score = 100;
    for (var i=0; i<actions.length; i++)
        {var newstate = simulate(actions[i],state,library);
        var newscore = mcs(role,newstate,level);
        if (newscore===0) {return 0};
        if (newscore<score) {score = newscore}};
    return score}
```


## Implementation

```
function montecarlo (state)
    {var total = 0;
        for (var i=0; i<count; i++)
        {total = total + depthcharge(state)};
    return total/count}
function depthcharge (state)
    {if (findterminalp(state,ruleset))
        {return findreward(role,state,ruleset)}*1;
    var actions = findlegals(state,library);
    if (actions.length===0) {return 0};
    var best = randomindex(actions.length);
    var newstate = simulate(actions[best],state,library);
    return depthcharge(newstate)}
```


## Problems and Features

Problems
Optimistic - opponent might not respect probabilities Does not utilize game structure in any useful way

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## Problems

Optimistic - opponent might not respect probabilities
Does not utilize game structure in any useful way

## Benefits

Fast because no branching in depth charges Small space because nothing stored in probes Provides guidance when other heuristics fail

Issues

## Incomplete Search



## Evaluation of Non-Terminal States



How do we evaluate non-terminal states?

## Choice of Depth



To what depth should we search?

## Variable Depth Search



Can we search different branches to different depths?

## Persistence



Can we preserve tree across moves or phases of ID?



