

Ordered Resolution

Unordered Propositional Resolution

Ordinary resolution is *unordered*. The rule can be applied to any literal in either clause.

$$\frac{\{\varphi_1, \dots, \chi, \dots, \varphi_m\} \quad \{\psi_1, \dots, \neg\chi, \dots, \psi_n\}}{\{\varphi_1, \dots, \varphi_m, \psi_1, \dots, \psi_n\}}$$

The more opportunities for resolution, the larger the search space.

Example

Negated Goal

$$\{\neg p_3, \neg q_3, \neg r_3\}$$

Premises

$\{p_3, \neg p_2\}$	$\{q_3, \neg q_2\}$	$\{r_3, \neg r_2\}$
$\{p_2, \neg p_1\}$	$\{q_2, \neg q_1\}$	$\{r_2, \neg r_1\}$
$\{p_1\}$	$\{q_1\}$	$\{r_1\}$

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Large Search Space

$\{\neg p_3, \neg q_3, \neg r_3\}$	$\{\neg p_2, \neg q_3, \neg r_3\}$	$\{\neg p_1, \neg q_3, \neg r_3\}$
$\{\neg p_3, \neg q_3, \neg r_2\}$	$\{\neg p_2, \neg q_3, \neg r_2\}$	$\{\neg p_1, \neg q_3, \neg r_2\}$
$\{\neg p_3, \neg q_3, \neg r_1\}$	$\{\neg p_2, \neg q_3, \neg r_1\}$	$\{\neg p_1, \neg q_3, \neg r_1\}$
$\{\neg p_3, \neg q_2, \neg r_3\}$	$\{\neg p_2, \neg q_2, \neg r_3\}$	$\{\neg p_1, \neg q_2, \neg r_3\}$
$\{\neg p_3, \neg q_2, \neg r_2\}$	$\{\neg p_2, \neg q_2, \neg r_2\}$	$\{\neg p_1, \neg q_2, \neg r_2\}$
$\{\neg p_3, \neg q_2, \neg r_1\}$	$\{\neg p_2, \neg q_2, \neg r_1\}$	$\{\neg p_1, \neg q_2, \neg r_1\}$
$\{\neg p_3, \neg q_1, \neg r_3\}$	$\{\neg p_2, \neg q_1, \neg r_3\}$	$\{\neg p_1, \neg q_1, \neg r_3\}$
$\{\neg p_3, \neg q_1, \neg r_2\}$	$\{\neg p_2, \neg q_1, \neg r_2\}$	$\{\neg p_1, \neg q_1, \neg r_2\}$
$\{\neg p_3, \neg q_1, \neg r_1\}$	$\{\neg p_2, \neg q_1, \neg r_1\}$	$\{\neg p_1, \neg q_1, \neg r_1\}$

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Intuition for Ordered Resolution

To derive empty clause, *every* literal must be eliminated.

$$\begin{array}{ccc} \{ \neg p_3, & \neg q_3, & \neg r_3 \} \\ | & | & | \\ \neg p_2 & \neg q_2 & \neg r_2 \\ | & | & | \\ \neg p_1 & \neg q_1 & \neg r_1 \end{array}$$

Idea: work on one literal till it is gone before starting to work on other literals.

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Clauses and Chains

A *clause* is a set of literals.

$$\{p, \neg q, \neg r\}$$

A *chain* is a sequence of literals

$$\langle p, \neg q, \neg r \rangle$$

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Ordered Propositional Resolution

$$\frac{\langle \chi, \varphi_1, \dots, \varphi_m \rangle \quad \langle \neg \chi, \psi_1, \dots, \psi_n \rangle}{\langle \varphi_1, \dots, \varphi_m, \psi_1, \dots, \psi_n \rangle}$$

Note the order of the literals in the resolvent. Either way works, but you must be consistent.

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Ordered Relational Resolution

$$\frac{\langle \varphi, \varphi_1, \dots, \varphi_m \rangle \quad \langle \neg \psi, \psi_1, \dots, \psi_n \rangle}{\langle \varphi_1 \tau, \dots, \varphi_m \tau, \psi_1, \dots, \psi_n \rangle \sigma}$$

where $\sigma = mgu(\varphi\tau, \psi)$

where τ is a variable renaming on φ

NB: Still need factoring.

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Smaller Search Space

1. $\langle p_3, \neg p_2 \rangle$
2. $\langle p_2, \neg p_1 \rangle$
3. $\langle p_1 \rangle$
4. $\langle q_3, \neg q_2 \rangle$
5. $\langle q_2, \neg q_1 \rangle$
6. $\langle q_1 \rangle$
7. $\langle r_3, \neg r_2 \rangle$
8. $\langle r_2, \neg r_1 \rangle$
9. $\langle r_1 \rangle$
10. $\langle \neg p_3, \neg q_3, \neg r_3 \rangle$
11. $\langle \neg p_2, \neg q_3, \neg r_3 \rangle$
12. $\langle \neg p_1, \neg q_3, \neg r_3 \rangle$
13. $\langle \neg q_3, \neg r_3 \rangle$
14. $\langle \neg q_2, \neg r_3 \rangle$
15. $\langle \neg q_1, \neg r_3 \rangle$
16. $\langle \neg r_3 \rangle$
17. $\langle \neg r_2 \rangle$
18. $\langle \neg r_1 \rangle$
19. $\langle \rangle$

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Does Not Play Well With Others

Ordered Resolution + Set of Support is not complete.

1. $\langle p, q \rangle$ Premise
2. $\langle \neg p, q \rangle$ Premise
3. $\langle \neg q \rangle$ Goal

Two Complete Answers and One Partial Answer:

Semi-Ordered Resolution

Contrapositives

Horn Restriction

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Semi-Ordered Resolution

$$\langle \varphi_1, \dots, \varphi_m \rangle$$
$$\langle \neg \psi, \psi_1, \dots, \psi_n \rangle$$

$$\langle \varphi_1 \tau, \dots, \varphi_m \tau, \psi_1, \dots, \psi_n \rangle \sigma$$

where $\sigma = mgu(\varphi\tau, \psi)$

where τ is a variable renaming on φ

NB : Still need factoring.

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Example

1. $\langle p, q \rangle$ Premise
2. $\langle \neg p, q \rangle$ Premise
3. $\langle \neg q \rangle$ Goal
4. $\langle p \rangle$ 1,3
5. $\langle \neg p \rangle$ 2,3
6. $\langle \rangle$ 4,5

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Contrapositives

A *contrapositive* of a chain is a permutation in which a different literal is placed at the front.

Chain: $\langle p, \neg q, \neg r \rangle$

Contrapositive: $\langle \neg q, p, \neg r \rangle$

Contrapositive: $\langle \neg r, p, \neg q \rangle$

The contrapositives of a chain are logically equivalent to the original chain.

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Example

1. $\langle p, q \rangle$ Premise
2. $\langle q, p \rangle$ Premise
3. $\langle \neg p, q \rangle$ Premise
4. $\langle q, \neg p \rangle$ Premise
5. $\langle \neg q \rangle$ Goal
6. $\langle p \rangle$ 2,5
7. $\langle \neg p \rangle$ 4,5
8. $\langle \rangle$ 6,7

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Horn Clauses and Horn Chains

A *Horn clause* is a clause containing at most one positive literal.

A *Horn chain* is a chain containing at most one positive literal.

Example: $\langle r, \neg p, \neg q \rangle$

Example: $\langle \neg p, \neg q, \neg r \rangle$

Example: $\langle p \rangle$

Non-Example: $\langle q, r, \neg p \rangle$

In Horn chains, positive literals are usually put first.

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Completeness

Metatheorem: If Δ consists entirely of Horn chains and all chains are ordered with positive literals, if any, first, then there is a resolution refutation of Δ if and only if there is a set of support refutation using ordered resolution.

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Example

- | | | |
|-----|-------------------------------------|---------|
| 1. | $\langle m \rangle$ | Premise |
| 2. | $\langle p, \neg m \rangle$ | Premise |
| 3. | $\langle q, \neg m \rangle$ | Premise |
| 4. | $\langle r, \neg p, \neg q \rangle$ | Premise |
| 5. | $\langle \neg r \rangle$ | Goal |
| 6. | $\langle \neg p, \neg q \rangle$ | 4,5 |
| 7. | $\langle \neg m, \neg q \rangle$ | 2,6 |
| 8. | $\langle \neg q \rangle$ | 1,7 |
| 9. | $\langle \neg m \rangle$ | 3,8 |
| 10. | $\langle \rangle$ | 1,9 |

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Example

- | | | |
|----|--|---------|
| 1. | $\langle p(\text{art}, \text{bob}) \rangle$ | Premise |
| 2. | $\langle p(\text{art}, \text{bud}) \rangle$ | Premise |
| 3. | $\langle p(\text{bob}, \text{cal}) \rangle$ | Premise |
| 4. | $\langle p(\text{bud}, \text{coe}) \rangle$ | Premise |
| 5. | $\langle g(x, z), \neg p(x, y), \neg p(y, z) \rangle$ | Premise |
| 6. | $\langle \neg g(\text{art}, \text{coe}) \rangle$ | Goal |
| 7. | $\langle \neg p(\text{art}, y), \neg p(y, \text{coe}) \rangle$ | 5,6 |
| 8. | $\langle \neg p(\text{bud}, \text{coe}) \rangle$ | 2,7 |
| 9. | $\langle \rangle$ | 4,8 |

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Input Resolution

An *input resolution* is one in which at least one of the parents is a member of the initial database (premise or goal).

Fcat: Input resolution is not complete.

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Linear Resolution

A linear resolution is one in which one of the parents is an input clause or an ancestor of the other clause.

More specifically, the resolution can be restricted to those in which the resolution involves the same literal in the parent that led to the child.

Metatheorem: Linear Resolution is complete!!

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Model Elimination

Model Elimination is a variant of Ordered Resolution that incorporates the Linearity Restriction in the definition of the rules of inference.

Using Model Elimination alone, it is possible to build a theorem prover that is sound and complete for all of Relational Logic.

Moreover, it works with the Set of Support strategy and the Input Restriction!!!

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Normal and Reduced Literals

Normal Literals:

$$\begin{array}{c} p \\ \neg q \end{array}$$

Reduced Literals:

$$\begin{array}{c} [p] \\ [\neg q] \end{array}$$

Chains:

$$\langle p, \neg q, [p], r \rangle$$

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Reduction

Reduction

$$\frac{\langle \varphi, \varphi_1, \dots, \varphi_m \rangle \quad \langle \psi, \psi_1, \dots, \psi_n \rangle}{\langle \varphi_1, \dots, \varphi_m, [\psi], \psi_1, \dots, \psi_n \rangle \sigma}$$

where $\sigma = mgu(\neg \varphi, \psi)$

Example

$$\frac{\langle g(x, z), \neg p(x, y), \neg p(y, z) \rangle \quad \langle \neg g(a, v), \neg h(v) \rangle}{\langle \neg p(a, y), \neg p(y, v), [\neg g(a, v)], \neg h(v) \rangle}$$

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Cancellation

Cancellation

$$\frac{\langle \varphi, \varphi_1, \dots, \varphi_m, [\psi], \psi_1, \dots, \psi_n \rangle}{\langle \varphi_1, \dots, \varphi_m, [\psi], \psi_1, \dots, \psi_n \rangle \sigma}$$

where $\sigma = mgu(\neg \varphi, \psi)$

Example

$$\frac{\langle g(x, b), \neg p(x, y), [\neg g(a, z)], \neg h(z) \rangle}{\langle \neg p(a, y), [\neg g(a, b)], \neg h(b) \rangle}$$

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Dropping

Dropping

$$\frac{\langle [\varphi], \varphi_1, \dots, \varphi_m \rangle}{\langle \varphi_1, \dots, \varphi_m \rangle}$$

Example

$$\frac{\langle [\neg g(a, v)], \neg h(v) \rangle}{\langle \neg h(v) \rangle}$$

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Example

1. $\langle p \rangle$ Premise
2. $\langle q \rangle$ Premise
3. $\langle r, \neg p, \neg q \rangle$ Premise
4. $\langle \neg r \rangle$ Goal
5. $\langle \neg p, \neg q, [\neg r] \rangle$ Reduction : 3, 4
6. $\langle [\neg p], \neg q, [\neg r] \rangle$ Reduction : 1, 5
7. $\langle \neg q, [\neg r] \rangle$ Dropping : 6
8. $\langle [\neg q], [\neg r] \rangle$ Reduction : 2, 7
9. $\langle [\neg r] \rangle$ Dropping : 8
10. $\langle \rangle$ Dropping : 9

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Example

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|-----|--|------------------|
| 1. | $\langle p, q \rangle$ | Premise |
| 2. | $\langle r, \neg p \rangle$ | Premise |
| 3. | $\langle \neg q, r \rangle$ | Premise |
| 4. | $\langle s \rangle$ | Premise |
| 5. | $\langle \neg r, \neg s \rangle$ | Goal |
| 6. | $\langle \neg p, [\neg r], \neg s \rangle$ | Reduction : 2,5 |
| 7. | $\langle q, [\neg p], [\neg r], \neg s \rangle$ | Reduction : 1,6 |
| 8. | $\langle r, [q], [\neg p], [\neg r], \neg s \rangle$ | Reduction : 3,7 |
| 9. | $\langle [q], [\neg p], [\neg r], \neg s \rangle$ | Cancellation : 8 |
| 10. | $\langle [\neg p], [\neg r], \neg s \rangle$ | Dropping : 9 |
| 11. | $\langle [\neg r], \neg s \rangle$ | Dropping : 10 |
| 12. | $\langle \neg s \rangle$ | Dropping : 11 |
| 13. | $\langle [\neg s] \rangle$ | Reduction : 4,12 |
| 14. | $\neg s$ | Dropping : 13 |

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No Forward Cancellation

- | | | |
|-----|---|--------------------------|
| 1. | $\langle p, q \rangle$ | Premise |
| 2. | $\langle \neg q, \neg r \rangle$ | Premise |
| 3. | $\langle r \rangle$ | Premise |
| 4. | $\langle \neg p, \neg q \rangle$ | Goal |
| 5. | $\langle q, [\neg p], \neg q \rangle$ | Reduction : 1, 4 |
| 6. | $\langle \neg r, [q], [\neg p], \neg q \rangle$ | Reduction : 2, 5 |
| 7. | $\langle [q], [\neg p], \neg q \rangle$ | Reduction : 3, 6 |
| 8. | $\langle [q], [\neg p] \rangle$ | Cancellation : 7 Wrong!! |
| 9. | $\langle [\neg p] \rangle$ | Dropping : 8 |
| 10. | $\langle \rangle$ | Dropping : 9 |

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Notes

Still need contrapositives or semi-ordered resolution.

Reduced literals unnecessary when inputs are Horn.

No need for factoring!!

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Summary

Ordered Resolution with Factoring

Complete without strategies

Incompatible with Set of Support

Incompatible with Input Restriction

Semi-Ordered Resolution and/or Contrapositives

Complete

Works with Set of Support

Incompatible with Input Restriction

Model Elimination

Complete

Works with Set of Support

Works with Input Restriction

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