Computational Logic | Lecture 12

Strategies

Michael Genesereth | Autumn 2007

Plan

First Lecture - Resolution Preliminaries
- Unification
- Relational Clausal Form

Second Lecture - Resolution Principle
- Resolution Principle and Factoring
- Resolution Theorem Proving

Third Lecture - Resolution Applications
- Theorem Proving
- Answer Extraction
- Residue

Fourth Lecture - Resolution Strategies
- Elimination Strategies (tautology elimination, subsumption, …)
- Restriction Strategies (ancestry filtering, set of support, …)

2

Proof

1. $\{p, q\}$ $\quad p \vee q$
2. $\{p, \neg q\}$ $\quad p \vee \neg q$
3. $\{\neg p, q\}$ $\quad \neg p \vee q$
4. $\{\neg p, \neg q\}$ $\quad \neg p \vee \neg q$
5. $\{p\}$ $\quad 1,2$
6. $\{\neg p\}$ $\quad 3,4$
7. $\{\}$ $\quad 5,6$

Poem

We shall not cease from exploration,
And the end of all our exploring
Will be to arrive where we started
And know the place for the first time.

T. S. Eliot

Proof as Produced by Two Finger Method

1.	$\{p, q\}$	$p \vee q$
2.	$\{p, \neg q\}$	$p \vee \neg q$
3.	$\{\neg p, q\}$	$\neg p \vee q$
4.	$\{\neg p, \neg q\}$	$\neg p \vee \neg q$
5.	$\{p\}$	1,2
6.	$\{q\}$	1,3
7.	$\{\neg q, q\}$	2,3
8.	$\{p, \neg p\}$	2,3
9.	$\{q, \neg q\}$	1,4
9.5	$\{p, \neg p\}$	1,4
10.	$\{\neg q\}$	2,4
11.	$\{\neg p\}$	3,4
12.	$\{q\}$	3,5
13.	$\{\neg q\}$	4,5
14.	$\{p\}$	2,6
15.	$\{\neg p\}$	4,6
16.	$\{p, q\}$	1,7
17.	$\{\neg q, p\}$	2,7
18.	$\{\neg p, q\}$	3,7
19.	$\{\neg q, \neg p\}$	4,7
20.	$\{q\}$	6,7

5

Proof (continued)

21.	$\{\neg q, q\}$	7,7
22.	$\{\neg q, q\}$	7,7
23.	$\{q, p\}$	1,8
24.	$\{\neg q, p\}$	2,8
25.	$\{\neg p, q\}$	3,8
26.	$\{\neg p, \neg q\}$	4,8
27.	$\{p\}$	5,8
28.	$\{\neg p, p\}$	8,8
29.	$\{\neg p, p\}$	8,8
30.	$\{p, q\}$	1,9
31.	$\{\neg q, p\}$	2,9
32.	$\{\neg p, q\}$	3,9
33.	$\{\neg q, \neg p\}$	4,9
34.	$\{q\}$	6,9
35.	$\{\neg q, q\}$	7,9
36.	$\{q, \neg q\}$	9,9
37.	$\{q, \neg q\}$	9,9
38.	$\{p\}$	1,10
39.	$\{\neg p\}$	3,10
40.	$\{\}$	6,10

6

Strategies

Elimination Strategies (Constraints on clauses):

- Identical Clause Elimination
- Pure Literal Elimination
- Tautology Elimination
- Subsumption Elimination

Restriction Strategies (Constraints on inferences):

- Unit Restriction
- Input Restriction
- Linear Restriction
- Set of Support Restriction

7

Identical Clause Elimination

Metatheorem: There is a resolution refutation of Δ if and only if there is a resolution refutation from Δ in which no clause occurs twice. (Obvious.)

Upshot: If you generate a clause that is already in the proof, do not include it again.

8

Proof With Identical Clause Elimination

1.	$\{p, q\}$	$p \vee q$
2.	$\{p, \neg q\}$	$p \vee \neg q$
3.	$\{\neg p, q\}$	$\neg p \vee q$
4.	$\{\neg p, \neg q\}$	$\neg p \vee \neg q$
5.	$\{p\}$	1,2
6.	$\{q\}$	1,3
7.	$\{\neg q, q\}$	2,3
8.	$\{p, \neg p\}$	2,3
9.	$\{q, \neg q\}$	1,4
10.	$\{\neg q\}$	2,4
11.	$\{\neg p\}$	3,4
12.	$\{\}$	6,10

9

Motivation for Tautology Elimination

1.	$\{p, q\}$	Premise
2.	$\{p, \neg p\}$	Premise
3.	$\{p, q\}$	1,2

10

Tautology Elimination

A *tautology* is a clause with a complementary pair of literals.

$$\{q, \neg q\}$$

$$\{p, q, r, \neg q\}$$

$$\{p(x), q(a,y), \neg q(a,y), r(z)\}$$

Metatheorem: There is a resolution refutation of Δ if and only if there is a resolution refutation from Δ with tautology elimination.

11

Proof with TE and ICE

1.	$\{p, q\}$	$p \vee q$
2.	$\{p, \neg q\}$	$p \vee \neg q$
3.	$\{\neg p, q\}$	$\neg p \vee q$
4.	$\{\neg p, \neg q\}$	$\neg p \vee \neg q$
5.	$\{p\}$	1,2
6.	$\{q\}$	1,3
7.	$\{\neg q\}$	2,4
8.	$\{\neg p\}$	3,4
9.	$\{\}$	6,7

12

Note

Non-Tautology:

$$\{p(x),\ \neg p(a)\}$$

Reason for Non-Example:

$$\{p(x),\ \neg p(a)\}$$
$$\{p(a)\}$$
$$\{\neg p(b)\}$$

13

Motivation for Subsumption

1. $\{p,q\}$ Premise
2. $\{p,q,r\}$ Premise
3. $\{q,r\}$ Premise
4. $\{\neg p\}$ Premise
5. $\{\neg q\}$ Premise
6. $\{\neg r\}$ Premise

14

Propositional Subsumption

A clause Φ *subsumes* Ψ if and only if Φ is a subset of Ψ.

Example: $\{p, q\}$ subsumes $\{p, q, r\}$

Theorem: There is a resolution refutation of Δ if and only if there is a resolution refutation from Δ with Propositional Subsumption.

15

Note

The resolution of two clauses sometimes produces a clause that subsumes one of its parents.

1.	$\{p\}$	Premise
2.	$\{\neg r, q\}$	Premise
3.	$\{r\}$	Premise
4.	$\{\neg p, \neg q, \neg r\}$	Premise
5.	$\{\neg q, \neg r\}$	1,4
6.	$\{\neg r\}$	2,5
7.	$\{\}$	3,6

16

Relational Subsumption

A relational clause Φ *subsumes* Ψ if and only if there is a substitution σ that, when applied to Φ, produces a clause $\Phi\sigma$ that is a subset of Ψ.

$$\{\neg p(a,b), q(c)\}$$
$$\{\neg p(x,y)\}$$

Why:

$\{\neg p(x,y)\}\{x \leftarrow a, y \leftarrow b\} = \{\neg p(a,b)\} \subseteq \{\neg p(a,b), q(c)\}$

Metatheorem: There is a resolution refutation of Δ if and only if there is a resolution refutation from Δ with subsumption.

17

Note

Non-Example:

$$\{\neg p(x,b), q(x)\}$$
$$\{\neg p(a,y)\}$$

Reason for Non-Example:

$$\{\neg p(x,b), q(x)\}$$
$$\{\neg p(a,y)\}$$
$$\{p(b,b)\}$$
$$\{\neg q(b)\}$$

18

Example for Pure Literal Elimination

1.	$\{p, q\}$	Premise
2.	$\{\neg p, r\}$	Premise
3.	$\{\neg q, r\}$	Premise
4.	$\{\neg q, s\}$	Premise
5.	$\{\neg r\}$	Goal

19

Pure Literal Elimination

A literal in a database is *pure* if and only if there is no complementary occurrence of the literal in the database.

A clause is *superfluous* if and only if it contains a pure literal.

Metatheorem: There is a resolution refutation of Δ if and only if there is a resolution refutation from Δ in which all superfluous clauses are removed.

20

Example

1.	$\{p, q\}$	Premise
2.	$\{\neg p, r\}$	Premise
3.	$\{\neg q, r\}$	Premise
4.	$\{\neg q, s\}$	Premise
5.	$\{\neg r\}$	Goal

21

Note

The removal of a superfluous clause may create new pure literals and new superfluous clauses.

1.	$\{p, q\}$	$p \vee q$
2.	$\{\neg p, r\}$	$p \Rightarrow r$
3.	$\{\neg q, r\}$	$q \Rightarrow r$
4.	$\{\neg q, s, t\}$	$q \Rightarrow s \vee t$
5.	$\{\neg r\}$	$\neg r$
6.	$\{\neg t\}$	$\neg t$

22

Strategies

Elimination Strategies (Constraints on clauses):

- Identical Clause Elimination
- Tautology Elimination
- Subsumption Elimination
- Pure Literal Elimination

Restriction Strategies (Constraints on inferences):

- Unit Restriction
- Input Restriction
- Linear Restriction
- Set of Support Restriction

Unit Restriction

A unit clause is a clause containing exactly one literal.

Examples:

$$\{p\}$$
$$\{\neg p\}$$

Non-Examples:

$$\{\}$$
$$\{p(x), q(a)\}$$

A *unit resolution* is one in which at least one of the parents is a unit clause.

Examples

1.	$\{p, q\}$		11.	$\{r\}$	3,6	21.	$\{r\}$	6,10
2.	$\{\neg p, r\}$		12.	$\{r, s\}$	4,6	22.	$\{\}$	5,11
3.	$\{\neg q, r\}$		13.	$\{q\}$	5,6			
4.	$\{\neg q, s\}$		14.	$\{r\}$	2,7			
5.	$\{\neg r\}$		15.	$\{p\}$	5,7			
6.	$\{q, r\}$	1,2	16.	$\{r, s\}$	2,8			
7.	$\{p, r\}$	1,3	17.	$\{q\}$	1,9			
8.	$\{p, s\}$	1,4	18.	$\{r\}$	7,9			
9.	$\{\neg p\}$	2,5	19.	$\{s\}$	8,9			
10.	$\{\neg q\}$	3,5	20.	$\{p\}$	1,10			

25

Incompleteness of Unit Resolution

1. $\{p, q\}$ $\quad p \vee q$
2. $\{p, \neg q\}$ $\quad p \vee \neg q$
3. $\{\neg p, q\}$ $\quad \neg p \vee q$
4. $\{\neg p, \neg q\}$ $\quad \neg p \vee \neg q$

26

Input Restriction

An *input resolution* is one in which at least one of the parents is a member of the initial database (premise or goal).

Input Restriction

1.	$\{p,q\}$		11.	$\{r\}$	3,6	21.	$\{r\}$	6,10
2.	$\{\neg p,r\}$		12.	$\{r,s\}$	4,6	22.	$\{\}$	5,11
3.	$\{\neg q,r\}$		13.	$\{q\}$	5,6			
4.	$\{\neg q,s\}$		14.	$\{r\}$	2,7			
5.	$\{\neg r\}$		15.	$\{p\}$	5,7			
6.	$\{q,r\}$	1,2	16.	$\{r,s\}$	2,8			
7.	$\{p,r\}$	1,3	17.	$\{q\}$	1,9			
8.	$\{p,s\}$	1,4	18.	$\{r\}$	7,9			
9.	$\{\neg p\}$	2,5	19.	$\{s\}$	8,9			
10.	$\{\neg q\}$	3,5	20.	$\{p\}$	1,10			

Incompleteness of Input Restriction

Theorem: Input resolution is not refutation complete.

1.	$\{p, q\}$	$p \vee q$
2.	$\{p, \neg q\}$	$p \vee \neg q$
3.	$\{\neg p, q\}$	$\neg p \vee q$
4.	$\{\neg p, \neg q\}$	$\neg p \vee \neg q$

Argument: In propositional case, parents of empty clause must both be units.

Curious Fact: Unit Resolution and Input Resolution work in exactly the same cases.

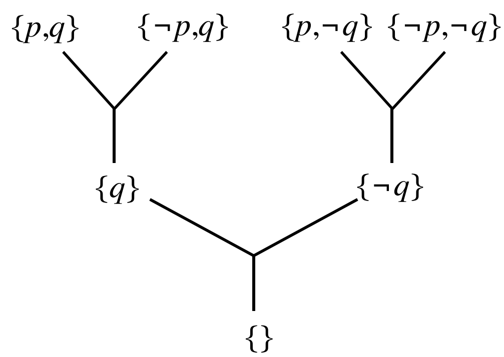
29

Motivation for Linear Restriction

1.	$\{p, q\}$	$p \vee q$
2.	$\{p, \neg q\}$	$p \vee \neg q$
3.	$\{\neg p, q\}$	$\neg p \vee q$
4.	$\{\neg p, \neg q\}$	$\neg p \vee \neg q$
5.	$\{p\}$	1,2
6.	$\{\neg p\}$	3,4
7.	$\{\}$	5,6

30

Elegant but Non-Linear Proof



31

Linear Restriction

A linear resolution is one in which one of the parents is an input clause or an ancestor of the other clause.

More specifically, the resolution can be restricted to those in which the resolution involves the same literal in the parent that led to the child.

Metatheorem: Linear Resolution is complete!!

32

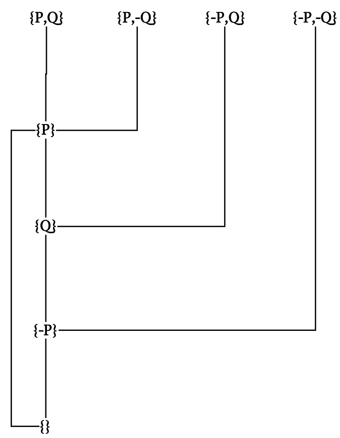
Non-Example

1.	$\{p, q\}$	$p \vee q$
2.	$\{p, \neg q\}$	$p \vee \neg q$
3.	$\{\neg p, q\}$	$\neg p \vee q$
4.	$\{\neg p, \neg q\}$	$\neg p \vee \neg q$
5.	$\{p\}$	1,2
6.	$\{\neg p\}$	3,4
7.	$\{\}$	5,6

Example

1.	$\{p, q\}$	$p \vee q$
2.	$\{p, \neg q\}$	$p \vee \neg q$
3.	$\{\neg p, q\}$	$\neg p \vee q$
4.	$\{\neg p, \neg q\}$	$\neg p \vee \neg q$
5.	$\{p\}$	1,2
6.	$\{q\}$	3,5
7.	$\{\neg p\}$	4,6
8.	$\{\}$	5,7

Linear Proof



35

Intuition for Set of Support

In many applications, there are many premises known to be satisfiable. Unsatisfiability comes from a subset (usually the negated goal).

Idea - avoid resolving premises with each other and save work.

36

Set of Support Restriction

Divide input clauses into *background data* and *set of support* in such a way that background data are known to be satisfiable. Typically, the set of support is set of clauses derived from goal. As each conclusion is produced, add it to the set of support.

A *set of support resolution* is one in which at least one parent is a member of the set of support. The result of a set of support resolution is added to the set of support.

Set of Support Resolution is complete!

37

Example

1.	$\{p,q\}$		11.	$\{r\}$	3,6	21.	$\{r\}$	6,10
2.	$\{\neg p,r\}$		12.	$\{r,s\}$	4,6	22.	$\{\}$	5,11
3.	$\{\neg q,r\}$		13.	$\{q\}$	5,6			
4.	$\{\neg q,s\}$		14.	$\{r\}$	2,7			
5.	$\{\neg r\}$	Goal	15.	$\{p\}$	5,7			
6.	$\{q,r\}$	1,2	16.	$\{r,s\}$	2,8			
7.	$\{p,r\}$	1,3	17.	$\{q\}$	1,9			
8.	$\{p,s\}$	1,4	18.	$\{r\}$	7,9			
9.	$\{\neg p\}$	2,5	19.	$\{s\}$	8,9			
10.	$\{\neg q\}$	3,5	20.	$\{p\}$	1,10			

38

Combinations of Strategies

All of the strategies presented here can be combined with each other without loss of completeness.

However, this is not true of all strategies. Some strategies do not combine well with others. As we shall see, even tautology elimination is not immune.

Let's be careful out there.

39