Computational Logic Lecture 11

Applications

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Plan

First Lecture - Resolution Preliminaries
- Unification
- Relational Clausal Form

Second Lecture - Resolution Principle
- Resolution Principle and Factoring
- Resolution Theorem Proving

Third Lecture - Resolution Applications
- Theorem Proving
- Answer Extraction
- Residue

Fourth Lecture - Resolution Strategies
- Elimination Strategies (tautology elimination, subsumption, …)
- Restriction Strategies (ancestry filtering, set of support, …)

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Determining Logical Entailment

To determine whether a set Δ of sentences logically entails a closed sentence φ, rewrite $\Delta \cup \{\neg\varphi\}$ in clausal form and try to derive the empty clause.

Example

Show that $(p(x) \Rightarrow q(x))$ and $p(a)$ logically entail $\exists z.q(z)$.

1.	$\{\neg p(x), q(x)\}$	Premise
2.	$\{p(a)\}$	Premise
3.	$\{\neg q(z)\}$	Goal
4.	$\{\neg p(z)\}$	1, 3
5.	$\{\}$	2, 4

Alternate Method

Basic Method: To determine whether a set Δ of sentences logically entails a closed sentence φ, rewrite $\Delta \cup \{\neg\varphi\}$ in clausal form and try to derive the empty clause.

Alternate Method: To determine whether a set Δ of sentences logically entails a closed sentence φ , rewrite $\Delta \cup \{\varphi \Rightarrow goal\}$ in clausal form and try to derive *goal*.

Intuition: The sentence $(\varphi \Rightarrow goal)$ is equivalent to the sentence $(\neg\varphi \vee goal)$.

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Example

Show that $(p(x) \Rightarrow q(x))$ and $p(a)$ logically entail $\exists z.q(z)$.

1.	$\{\neg p(x), q(x)\}$	$p(x) \Rightarrow q(x)$
2.	$\{p(a)\}$	$p(a)$
3.	$\{\neg q(z), goal\}$	$\exists z.q(z) \Rightarrow goal$
4.	$\{\neg p(z), goal\}$	$1, 3$
5.	$\{goal\}$	$2, 4$

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Answer Extraction Method

Alternate Method for Logical Entailment: To determine whether a set Δ of sentences logically entails a closed sentence φ , rewrite $\Delta \cup \{\varphi \Rightarrow goal\}$ in clausal form and try to derive *goal*.

Method for Answer Extraction: To get values for free variables $\nu_1,\ldots,\nu_n$ in φ for which Δ logically entails φ, rewrite $\Delta \cup \{\varphi \Rightarrow goal(\nu_1,\ldots,\nu_n)\}$ in clausal form and try to derive $goal(\nu_1,\ldots,\nu_n)$.

Intuition: The sentence $(q(z) \Rightarrow goal(z))$ says that, whenever, z satisfies q, it satisfies the "goal".

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Example

Given $(p(x) \Rightarrow q(x))$ and $p(a)$, find a term τ such that $q(\tau)$ is true.

1.	$\{\neg p(x), q(x)\}$	$p(x) \Rightarrow q(x)$
2.	$\{p(a)\}$	$p(a)$
3.	$\{\neg q(z), goal(z)\}$	$q(z) \Rightarrow goal(z)$
4.	$\{\neg p(z), goal(z)\}$	1, 3
5.	$\{goal(a)\}$	2, 4

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Example

Given ($p(x) \Rightarrow q(x)$) and $p(a)$ and $p(b)$, find a term τ such that $q(\tau)$ is true.

1.	$\{\neg p(x), q(x)\}$	$p(x) \Rightarrow q(x)$
2.	$\{p(a)\}$	$p(a)$
3.	$\{p(b)\}$	$p(b)$
4.	$\{\neg q(z), goal(z)\}$	$q(z) \Rightarrow goal(z)$
5.	$\{\neg p(z), goal(z)\}$	1,3
6.	$\{goal(a)\}$	2,5
7.	$\{goal(b)\}$	3,5

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Example

Given ($p(x) \Rightarrow q(x)$) and ($p(a) \vee p(b)$), find a term τ such that $q(\tau)$ is true.

1.	$\{\neg p(x), q(x)\}$	$p(x) \Rightarrow q(x)$
2.	$\{p(a), p(b)\}$	$p(a) \vee p(b)$
3.	$\{\neg q(z), goal(z)\}$	$q(z) \Rightarrow goal(z)$
4.	$\{\neg p(z), goal(z)\}$	1,3
5.	$\{p(b), goal(a)\}$	2,4
6.	$\{goal(a), goal(b)\}$	4,5

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Kinship

Art is the parent of Bob and Bud.
Bob is the parent of Cal and Coe.
A grandparent is a parent of a parent.

$$p(art,bob)$$
$$p(art,bud)$$
$$p(bob,cal)$$
$$p(bob,coe)$$
$$p(x,y) \wedge p(y,z) \Rightarrow g(x,z)$$

Is Art the Grandparent of Coe?

1.	$\{p(art,bob)\}$	$p(art,bob)$
2.	$\{p(art,bud)\}$	$p(art,bud)$
3.	$\{p(bob,cal)\}$	$p(bob,cal)$
4.	$\{p(bob,coe)\}$	$p(bob,coe)$
5.	$\{\neg p(x,y), \neg p(y,z), g(x,z)\}$	$p(x,y) \wedge p(y,z) \Rightarrow g(x,z)$
6.	$\{\neg g(art,coe), goal\}$	$g(art,coe) \Rightarrow goal$
7.	$\{\neg p(art,y), \neg p(y,coe), goal\}$	5,6
8.	$\{\neg p(bob,coe), goal\}$	1,7
9.	$\{goal\}$	4,8

Who is the Grandparent of Coe?

1.	$\{p(art,bob)\}$	$p(art,bob)$
2.	$\{p(art,bud)\}$	$p(art,bud)$
3.	$\{p(bob,cal)\}$	$p(bob,cal)$
4.	$\{p(bob,coe)\}$	$p(bob,coe)$
5.	$\{\neg p(x,y), \neg p(y,z), g(x,z)\}$	$p(x,y) \wedge p(y,z) \Rightarrow g(x,z)$
6.	$\{\neg g(x,coe), goal(x)\}$	$g(x,coe) \Rightarrow goal(x)$
7.	$\{\neg p(x,y), \neg p(y,coe), goal(x)\}$	5,6
8.	$\{\neg p(bob,coe), goal(art)\}$	1,7
9.	$\{goal(art)\}$	4,8

Who Are the Grandchildren of Art?

1.	$\{p(art,bob)\}$	$p(art,bob)$
2.	$\{p(art,bud)\}$	$p(art,bud)$
3.	$\{p(bob,cal)\}$	$p(bob,cal)$
4.	$\{p(bob,coe)\}$	$p(bob,coe)$
5.	$\{\neg p(x,y), \neg p(y,z), g(x,z)\}$	$p(x,y) \wedge p(y,z) \Rightarrow g(x,z)$
6.	$\{\neg g(art,z), goal(z)\}$	$g(art,z) \Rightarrow goal(z)$
7.	$\{\neg p(art,y), \neg p(y,z), goal(z)\}$	5,6
8.	$\{\neg p(bob,z), goal(z)\}$	1,7
9.	$\{\neg p(bud,z), goal(z)\}$	2,7
10.	$\{goal(cal)\}$	3,8
11.	$\{goal(coe)\}$	4,8

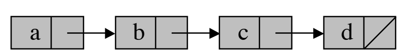
People and their Grandchildren?

1.	$\{p(art,bob)\}$	$p(art,bob)$
2.	$\{p(art,bud)\}$	$p(art,bud)$
3.	$\{p(bob,cal)\}$	$p(bob,cal)$
4.	$\{p(bob,coe)\}$	$p(bob,coe)$
5.	$\{\neg p(x,y), \neg p(y,z), g(x,z)\}$	$p(x,y) \wedge p(y,z) \Rightarrow g(x,z)$
6.	$\{\neg g(x,z), goal(x,z)\}$	$g(x,z) \Rightarrow goal(x,z)$
7.	$\{\neg p(x,y), \neg p(y,z), goal(x,z)\}$	5,6
8.	$\{\neg p(bob,z), goal(art,z)\}$	1,7
9.	$\{\neg p(bud,z), goal(art,z)\}$	2,7
10.	$\{goal(art,cal)\}$	3,8
11.	$\{goal(art,coe)\}$	4,8

Variable Length Lists

Example

$[a,b,c,d]$



Representation as Term

$cons(a,cons(b,cons(c,cons(d,nil))))$

Shorthand

(a . (b . (c . (d . nil))))

Shorthand

$[a,b,c,d]$

List Membership

Membership axioms:

member(u, *cons*(u, x))

member(u, *cons*(v, y)) ⇐ *member*(u, y)

Membership Clauses:

{*member*(u, u . x)}

{*member*(u, v . y), ¬*member*(u, y)}

Answer Extraction for *member*(w, [a, b])

{¬*member*(w, *a.b.c. nil*),goal(w)}

{*goal*(a)}

{¬*member*(w, *b.c. nil*)}

{*goal*(b)}

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List Concatenation

Concatenation Axioms:

append(*nil*,y,y)

append($w.x$, y, $w.z$) ⇐ *append*(x, y, z)

Concatenation Clauses:

{*append*(*nil*,y,y)}

{*append*($w.x$, y, $w.z$), ¬*append*(x, y, z)}

Answer Extraction for *append*([a,b],[c,d],z):

{¬*append(a.b.nil, c.d.nil, z*), *goal*(z)}

{¬*append(b.nil, c.d.nil, z1*), *goal*(*a.z1*)}

{¬*append(nil, c.d.nil, z2*), *goal*(*a.b.z2*)}

{*goal*(*a.b.c.d.nil*)}

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List Reversal

Reversal Example:

$$reverse([a,b,c,d], [d,c,b,a])$$

Reversal Axioms:

$$reverse(x, y) \Leftarrow reverse2(x, nil, y)$$
$$reverse2(nil, y, y)$$
$$reverse2(w.x, y, z) \Leftarrow reverse2(x, w.y, z)$$

Answer Extraction for $reverse([a,b,c,d],z)$:

$$\{\neg reverse(a.b.c.d.nil, z), goal(z)\}$$
$$\ldots$$
$$\{goal(d.c.b.a.nil)\}$$

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Natural Language Processing

Grammar:

S → NP VP
NP → Noun
NP → Noun and Noun
VP → Verb NP
Noun → Harry | Ralph | Mary
Verb → hate hates

Logical Form:

$S(z) \Leftarrow NP(x) \wedge VP(y) \wedge append(x,y,z)$
$NP(z) \Leftarrow Noun(z)$
$NP(z) \Leftarrow NP(x) \wedge NP(y) \wedge append(x,\text{and},x1) \wedge append(x1,y,z)$
$VP(z) \Leftarrow Verb(z)$
$Noun(\text{Harry}) \wedge Noun(\text{Ralph}) \wedge Noun(\text{Mary})$
$Verb(\text{hate}) \wedge Verb(\text{hates})$

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Sentence Generation as Answer Extraction

{¬*S*(*z*),*goal*(z)}
{¬*NP*(x),¬*VP*(*y*),¬*append*(*x,y,z*),*goal*(*z*)}
{¬*VP*(*y*),¬*append*(*Harry,y,z*),*goal*(*z*)}
{¬*Verb*(*y1*),¬*NP*(*y2*),¬*append*(*y1,y2,y*),¬*append*(Harry,*y,z*),*goal*(*z*)}
{¬*NP*(*y2*),¬*append*(hates,y2,y),¬*append*(Harry,*y*,*z*),*goal*(*z*)}
{¬*Noun*(y2),¬*append*(hates,*y2*,*y*),¬*append*(Harry,*y*,*z*),*goal*(*z*)}
{¬*append*(hates,Mary,*y*),¬*append*(Harry,*y*,*z*),*goal*(*z*)}
{¬*append*(Harry,hates Mary,*z*),*goal*(*z*)}
{*goal*(Harry hates Mary)}

{*goal*(Harry and Ralph hate Mary)}
{*goal*(Harry hate Mary)}
{*goal*(Harry and Ralph hates Mary)}

How can we enforce subject-verb number agreement?

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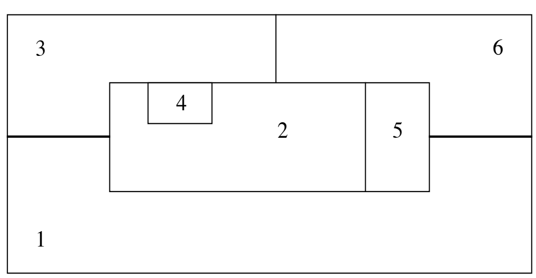
Map Coloring Problem

Color the map with the 4 colors red, green, blue, and purple such that no two regions have the same color.



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Acceptable Neighbors

$n(red, green)$ $n(blue, red)$
$n(red, blue)$ $n(blue, green)$
$n(red, purple)$ $n(blue, purple)$

$n(green, red)$ $n(purple, red)$
$n(green, blue)$ $n(purple, blue)$
$n(green, purple)$ $n(purple, green)$

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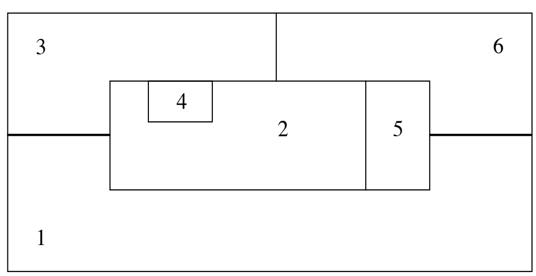
Question



$$n(x_1, x_2) \wedge n(x_1, x_3) \wedge n(x_1, x_5) \wedge n(x_1, x_6)$$
$$\wedge\, n(x_2, x_3) \wedge n(x_2, x_4) \wedge n(x_2, x_5) \wedge n(x_2, x_6)$$
$$\wedge\, n(x_3, x_4) \wedge n(x_3, x_6) \wedge n(x_5, x_6)$$
$$\Rightarrow goal(x_1, x_2, x_3, x_4, x_5, x_6)$$

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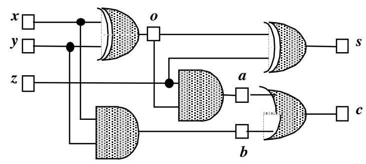
Circuit Simulation

Given inputs to circuit, determine outputs.



Behavior:

$o \Leftrightarrow (x \wedge \neg y) \vee (\neg x \wedge y)$

$a \Leftrightarrow z \wedge o$

$b \Leftrightarrow x \wedge y$

$s \Leftrightarrow (o \wedge \neg z) \vee (\neg o \wedge z)$

$c \Leftrightarrow a \vee b$

Inputs:

$x \wedge y \wedge \neg z$

Output:

c

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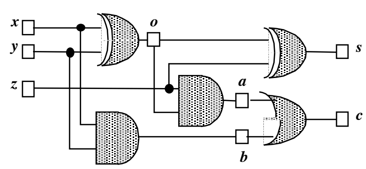
Circuit Simulation

Given inputs to circuit, determine outputs.



Behavior:

$o \Leftrightarrow (x \wedge \neg y) \vee (\neg x \wedge y)$

$a \Leftrightarrow z \wedge o$

$b \Leftrightarrow x \wedge y$

$s \Leftrightarrow (o \wedge \neg z) \vee (\neg o \wedge z)$

$c \Leftrightarrow a \vee b$

Desired Output:

c

Possible Inputs:

$x \wedge y$

$\neg x \wedge y \wedge z$

$x \wedge \neg y \wedge z$

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Residue

Problem: Given the data below, find conditions in terms of m, n, o, and a under which $s(a)$ is true.

$$p(x) \wedge q(x) \Rightarrow s(x)$$
$$m(x) \Rightarrow p(x)$$
$$n(x) \Rightarrow q(x)$$
$$o(x) \Rightarrow r(x)$$

Residue:

$$m(a) \wedge n(a)$$

In other words:

$$m(a) \wedge n(a) \Rightarrow s(a)$$

Method

To obtain the residue for φ given Δ in terms of $\rho_1,\ldots,\rho_n$, rewrite $\Delta \cup \{\neg\varphi\}$ in clausal form and try to derive a clause with constants restricted to $\rho_1,\ldots,\rho_n$. The residue is the negation of any clause so derived.

Example

1.	$\{\neg p(x), \neg q(x), s(x)\}$	$p(x) \wedge q(x) \Rightarrow s(x)$
2.	$\{\neg m(x), p(x)\}$	$m(x) \Rightarrow p(x)$
3.	$\{\neg n(x), q(x)\}$	$n(x) \Rightarrow q(x)$
4.	$\{\neg o(x), r(x)\}$	$o(x) \Rightarrow r(x)$
5.	$\{\neg s(a)\}$	$\neg s(a)$
6.	$\{\neg p(a), \neg q(a)\}$	1,5
7.	$\{\neg m(a), \neg q(a)\}$	2,6
8.*	$\{\neg m(a), \neg n(a)\}$	3,7

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Multiple Residues

1.	$\{\neg p(x), \neg q(x), s(x)\}$	$p(x) \wedge q(x) \Rightarrow s(x)$
2.	$\{\neg r(x), s(x)\}$	$r(x) \Rightarrow s(x)$
3.	$\{\neg m(x), p(x)\}$	$m(x) \Rightarrow p(x)$
4.	$\{\neg n(x), q(x)\}$	$n(x) \Rightarrow q(x)$
5.	$\{\neg o(x), r(x)\}$	$o(x) \Rightarrow r(x)$
6.	$\{\neg s(x)\}$	$\neg s(x)$
7.	$\{\neg p(x), \neg q(x)\}$	1,6
8.	$\{\neg m(x), \neg q(x)\}$	3,7
9. *	$\{\neg m(x), \neg n(x)\}$	4,8
10.	$\{\neg r(x)\}$	2,6
11.*	$\{\neg o(x)\}$	5,10

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Databases

p

art	*bob*
art	*bud*
amy	*bob*
amy	*bud*
bob	*cal*
bob	*coe*
bea	*cal*
bea	*coe*

male

art
bob
bud
cal

female

amy
bea
coe

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Databases as Ground Atomic Sentences

art	*bob*
art	*bud*
amy	*bob*
amy	*bud*
bob	*cal*
bob	*coe*
bea	*cal*
bea	*coe*

$p(art, bob)$
$p(art, bud)$
$p(amy, bob)$
$p(amy, bud)$
$p(bob, cal)$
$p(bob, coe)$
$p(bea, cal)$
$p(bea, coe)$

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Database Views

Father: $p(x,y) \wedge male(x) \Rightarrow f(x,y)$

Mother: $p(x,y) \wedge female(x) \Rightarrow m(x,y)$

Grandfather: $f(x,y) \wedge p(y,z) \Rightarrow gf(x,z)$

Grandmother: $m(x,y) \wedge p(y,z) \Rightarrow gm(x,z)$

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Database Query Planning

Query: $gf(u,v)$
Tables: $male, female, p.$

1.	$\{\neg p(x,y), \neg male(x), f(x,y)\}$	$p(x,y) \wedge male(x) \Rightarrow f(x,y)$
2.	$\{\neg p(x,y), \neg female(x), m(x,y)\}$	$p(x,y) \wedge female(x) \Rightarrow m(x,y)$
3.	$\{\neg f(x,y), \neg p(y,z), gf(x,z)\}$	$f(x,y) \wedge p(y,z) \Rightarrow gf(x,y)$
4.	$\{\neg m(x,y), \neg p(y,z), gm(x,z)\}$	$m(x,y) \wedge p(y,z) \Rightarrow gm(x,y)$
5.	$\{\neg gf(u,v)\}$	$\neg gf(u,v)$
6.	$\{\neg f(u,y), \neg p(y,v)\}$	3,5
7.*	$\{\neg p(u,y), \neg male(u), \neg p(y,v)\}$	1,6

Reformulated Query: $p(u,y) \wedge male(u) \wedge p(y,v)$

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Optimizations

Non-Optimal Query: Better Query:

Non-Optimal Query:	Better Query:
$p(x,y) \wedge p(y,z) \wedge p(y,x)$	*False*
$f(x,y) \wedge p(y,z) \wedge p(x,y)$	$f(x,y) \wedge p(y,z)$
$f(x,y) \vee m(x,y) \vee p(x,y)$	$p(x,y)$

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Constraints

Parenthood is antisymmetric:

$$p(x,y) \Rightarrow \neg p(y,x)$$

Fathers are parents:

$$f(x,y) \Rightarrow p(x,y)$$

Mothers are parents:

$$m(x,y) \Rightarrow p(x,y)$$

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Conjunction Elimination

Example: $p(x,y) \wedge p(y,z) \wedge p(y,x) \quad \rightarrow \quad \mathit{False}$

Constraint: $p(x,y) \Rightarrow \neg p(y,x) \qquad \{\neg p(x,y), \neg p(y,x)\}$

Technique: Check conjunction for contradiction. If found, delete the entire conjunction.

Clausal Form of $\exists x.\exists y.\exists z.(p(x,y) \wedge p(y,z) \wedge p(y,x))$:

$\{p(a,b)\}$
$\{p(b,c)\}$
$\{p(b,a)\}$

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Conjunctive Minimization

Example: $f(x,y) \wedge p(y,z) \wedge p(x,y) \quad \rightarrow \quad f(x,y) \wedge p(y,z)$

Constraint: $f(x,y) \Rightarrow p(x,y) \qquad \{\neg f(x,y), p(x,y)\}$

Technique: Check if conjunct is implied by others. If so, delete.

$f(x,y) \wedge p(y,z) \Rightarrow p(x,y)$
$f(x,y) \wedge p(x,y) \Rightarrow p(y,z)$
$p(y,z) \wedge p(x,y) \Rightarrow f(x,y)$

Clausal Form of $\neg \forall x.\forall y.\forall z.(f(x,y) \wedge p(y,z) \Rightarrow p(x,y))$:

$\{f(a,b)\}$
$\{p(b,c)\}$
$\{\neg p(a,b)\}$

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Disjunctive Minimization

Example: $f(x,y) \vee m(x,y) \vee p(x,y) \quad \rightarrow \quad p(x,y)$

Constraints: $f(x,y) \Rightarrow p(x,y) \qquad \{\neg f(x,y), p(x,y)\}$

Technique: Check if disjunct logically entails others.
If so, delete the disjunct.

$f(x,y) \Rightarrow m(x,y) \vee p(x,y)$
$m(x,y) \Rightarrow f(x,y) \vee p(x,y)$
$p(x,y) \Rightarrow f(x,y) \vee m(x,y)$

Clausal Form of $\neg\forall x.\forall y.\forall z.(f(x,y) \Rightarrow m(x,y) \vee p(x,y))$:
$\{f(a,b)\}$
$\{\neg m(a,b)\}$
$\{\neg p(a,b)\}$

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Recursion

Suppose we are given three tables *p*, *male*, and *female*.

The ancestor relation *a* is the transitive closure of the parent relation *p*.

$$p(x,y) \Rightarrow a(x,y)$$
$$a(x,y) \wedge a(y,z) \Rightarrow a(x,z)$$

How do we rewrite $a(u,v)$ in terms of p?

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Recursion

1.		$\{\neg p(x,y), a(x,y)\}$	$p(x,y) \Rightarrow a(x,y)$
2.		$\{\neg a(x,y), \neg a(y,z), a(x,z)\}$	$a(x,y) \wedge a(y,z) \Rightarrow a(x,z)$
3.		$\{\neg a(u,v)\}$	$\neg a(u,v)$
4.	*	$\{\neg p(u,v)\}$	1,3
5.		$\{\neg a(u,y), \neg a(y,v)\}$	2,3
6.		$\{\neg p(u,y), \neg a(y,v)\}$	1,4
7.	*	$\{\neg p(u,y), \neg p(y,v)\}$	1,5
8.		$\{\neg a(u,w), \neg a(w,y), \neg a(y,v)\}$	2,5
9.		$\{\neg p(u,w), \neg a(w,y), \neg a(y,v)\}$	1,8
10.		$\{\neg p(u,w), \neg p(w,y), \neg a(y,v)\}$	1,9
11.	*	$\{\neg p(u,w), \neg p(w,y), \neg p(y,v)\}$	1,10

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Recursion

Infinitely many residues!

$p(u,v)$

$p(u,y) \wedge p(y,v)$

$p(u,w) \wedge p(w,y) \wedge p(y,v)$

...

Solution: Include recursive relations among the residue relations. Let database handle recursive queries.

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