Computational Logic — Lecture 10

Resolution Theorem Proving

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Plan

First Lecture - Resolution Preliminaries
- Unification
- Relational Clausal Form

Second Lecture - Resolution Principle
- Resolution Principle and Factoring
- Resolution Theorem Proving

Third Lecture - Resolution Applications
- Theorem Proving
- Answer Extraction
- Reduction

Fourth Lecture - Resolution Strategies
- Elimination Strategies (tautology elimination, subsumption, …)
- Restriction Strategies (ancestry filtering, set of support, …)

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Propositional Resolution

$$\frac{\begin{array}{l}\{\varphi_1,\ldots,\varphi,\ldots,\varphi_m\}\\ \{\psi_1,\ldots,\neg\varphi,\ldots,\psi_n\}\end{array}}{\{\varphi_1,\ldots,\varphi_m,\psi_1,\ldots,\psi_n\}}$$

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Relational Resolution I

$$\frac{\begin{array}{l}\{\varphi_1,\ldots,\quad\varphi,\ldots,\varphi_m\}\\ \{\psi_1,\ldots,\neg\psi,\ldots,\psi_n\}\end{array}}{\{\varphi_1,\ldots,\varphi_m,\psi_1,\ldots,\psi_n\}\sigma}$$

where $\sigma = mgu(\varphi,\psi)$

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Example

$$\begin{array}{l} \{\ \ p(a,y), r(y)\} \\ \{\neg p(x,b)\} \\ \hline \{r(y)\}\{x \leftarrow a, y \leftarrow b\} \\ \{r(b)\} \end{array}$$

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Example

$$\begin{array}{l} \{\ \ p(a,y), \qquad r(y)\} \\ \{\neg p(x, f(x)), q(g(x))\} \\ \hline \{r(y), q(g(x))\}\{x \leftarrow a, y \leftarrow f(a)\} \\ \{r(f(a)), q(g(a))\} \end{array}$$

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Example

Everybody loves somebody. Everybody loves a lover. Show that everybody loves everybody.

$$\forall x.\exists y.loves(x,y)$$
$$\forall u.\forall v.\forall w.(loves(v,w) \Rightarrow loves(u,v))$$
$$\neg \forall x.\forall y.loves(x,y)$$

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Example (continued)

$$\forall x.\exists y.loves(x,y)$$
$$\forall u.\forall v.\forall w.(loves(v,w) \Rightarrow loves(u,v))$$
$$\neg \forall x.\forall y.loves(x,y)$$

$$\{loves(x,f(x))\}$$
$$\{\neg loves(v,w), loves(u,v)\}$$
$$\{\neg loves(jack,jill)\}$$

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Example (concluded)

1.	$\{loves(x, f(x))\}$	Premise
2.	$\{\neg loves(v,w), loves(u,v)\}$	Premise
3.	$\{\neg loves(jack, jill)\}$	Goal
4.	$\{loves(u,x)\}$	1,2
5.	$\{\}$	4,3

Harry and Ralph

Every horse can outrun every dog. Some greyhound can outrun every rabbit. Show that every horse can outrun every rabbit.

$$\forall x.\forall y.(h(x) \wedge d(y) \Rightarrow f(x,y))$$
$$\exists y.(g(y) \wedge \forall z.(r(z) \Rightarrow f(y,z)))$$
$$\forall y.(g(y) \Rightarrow d(y))$$
$$\forall x.\forall y.\forall z.(f(x,y) \wedge f(y,z) \Rightarrow f(x,z))$$
$$\neg \forall x.\forall y.(h(x) \wedge r(y) \Rightarrow f(x,y))$$

Harry and Ralph (continued)

$\forall x.\forall y.(h(x) \wedge d(y) \Rightarrow f(x,y))$

$\exists y.(g(y) \wedge \forall z.(r(z) \Rightarrow f(y,z)))$

$\forall y.(g(y) \Rightarrow d(y))$

$\forall x.\forall y.\forall z.(f(x,y) \wedge f(y,z) \Rightarrow f(x,z))$

$\{\neg h(x), \neg d(y), f(x,y)\}$

$\{g(greg)\}$

$\{\neg r(z), f(greg,z)\}$

$\{\neg g(y), d(y)\}$

$\{\neg f(x,y), \neg f(y,z), f(x,z)\}$

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Harry and Ralph (continued)

$\neg \forall x.\forall y.(h(x) \wedge r(y) \Rightarrow f(x,y))$

$\{h(harry)\}$

$\{r(ralph)\}$

$\{\neg f(harry,ralph)\}$

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Harry and Ralph (concluded)

1. $\{\neg h(x), \neg d(y), f(x,y)\}$
2. $\{g(greg)\}$
3. $\{\neg r(z), f(greg,z)\}$
4. $\{\neg g(y), d(y)\}$
5. $\{\neg f(x,y), \neg f(y,z), f(x,z)\}$
6. $\{h(harry)\}$
7. $\{r(ralph)\}$
8. $\{\neg f(harry,ralph)\}$
9. $\{d(greg)\}$
10. $\{\neg d(y), f(harry,y)\}$
11. $\{f(harry,greg)\}$
12. $\{f(greg,ralph)\}$
13. $\{\neg f(greg,z), f(harry,z)\}$
14. $\{f(harry,ralph)\}$
15. $\{\}$

Example

Given:

$$\exists x.\forall y.(p(x,y) \Leftrightarrow q(x,y))$$
$$\forall x.\exists y.(p(x,y) \vee q(x,y))$$

Prove:

$$\exists x.\exists y.(p(x,y) \wedge q(x,y))$$

Example (continued)

$\exists x.\forall y.(p(x,y) \Leftrightarrow q(x,y))$

$\exists x.\forall y.((\neg p(x,y) \vee q(x,y)) \wedge (p(x,y) \vee \neg q(x,y)))$

$(\neg p(a,y) \vee q(a,y)) \wedge (p(a,y) \vee \neg q(a,y))$

$\{\neg p(a,y), q(a,y)\}$

$\{p(a,y), \neg q(a,y)\}$

$\forall x.\exists y.(p(x,y) \vee q(x,y))$

$p(x, f(x)) \vee q(x, f(x))$

$\{p(a,f(x)), q(a,f(x))\}$

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Example (continued)

Negate the goal:

$\exists x.\exists y.(p(x,y) \wedge q(x,y)) \quad \rightarrow \quad \neg\exists x.\exists y.(p(x,y) \wedge q(x,y))$

Convert to Clausal Form:

$\neg\exists x.\exists y.(p(x,y) \wedge q(x,y))$

$\forall x.\forall y.\neg(p(x,y) \wedge q(x,y))$

$\forall x.\forall y.(\neg p(x,y) \vee \neg q(x,y))$

$\neg p(x,y) \vee \neg q(x,y)$

$\{\neg p(x,y), \neg q(x,y)\}$

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Example (concluded)

1. $\{\neg p(a,y), q(a,y)\}$	Premise
2. $\{p(a,y), \neg q(a,y)\}$	Premise
3. $\{p(x, f(x)), q(x, f(x))\}$	Premise
4. $\{\neg p(x,y), \neg q(x,y)\}$	Negated Goal
5. $\{q(a, f(a))\}$	1, 3
6. $\{p(a, f(a))\}$	2, 3
7. $\{\neg p(a, f(a))\}$	4, 5
8. $\{\}$	6, 7

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Example

$$\forall x.p(x) \Rightarrow \forall y.p(y)$$

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Example (continued)

	$\neg(\forall x.p(x) \Rightarrow \forall y.p(y))$
I:	$\neg(\neg\forall x.p(x) \vee \forall y.p(y))$
N:	$\neg\neg\forall x.p(x) \wedge \neg\forall y.p(y)$
	$\forall x.p(x) \wedge \exists y.\neg p(y)$
S:	$\forall x.p(x) \wedge \exists y.\neg p(y)$
E:	$\forall x.p(x) \wedge \neg p(a)$
A:	$p(x) \wedge \neg p(a)$
D:	$p(x) \wedge \neg p(a)$
O:	$\{p(x)\}$ and $\{\neg p(a)\}$

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Example (concluded)

Resolution:

1. $\{p(x)\}$	Premise
2. $\{\neg p(a)\}$	Premise
3. $\{\}$	1,2 $\{x \leftarrow a\}$

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Problem

$$\frac{\begin{array}{l}\{\ \ p(a,x)\} \\ \{\neg p(x,b)\}\end{array}}{\text{Failure}}$$

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Relational Resolution II

$$\frac{\begin{array}{l}\{\varphi_1,\ldots,\quad \varphi,\ldots,\varphi_m\} \\ \{\psi_1,\ldots,\neg\,\psi,\ldots,\psi_n\}\end{array}}{\{\varphi_1\tau,\ldots,\varphi_m\tau,\psi_1,\ldots,\psi_n\}\sigma}$$

where $\sigma = mgu(\varphi\tau,\psi)$

where τ is a variable renaming on φ

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Example

$\{\ \ p(a,x)\}$	$\{\ \ p(a,y)\}$
$\{\neg p(x,b)\}$	$\{\neg p(x,b)\}$
Failure	$\{\}\{x \leftarrow a, y \leftarrow b\}$

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Solution With Repeated Renaming

1.	$\{r(a,b,u1)\}$	Premise
2.	$\{r(b,c,u2)\}$	Premise
3.	$\{r(c,d,u3)\}$	Premise
4.	$\{r(x,z,f(v)),\neg r(x,y,f(f(v))),\neg r(y,z,f(f(v)))\}$	Premise
5.	$\{\neg r(a,d,w)\}$	Goal
6.	$\{\neg r(a,y6,f(f(v6))),\neg r(y6,d,f(f(v6)))\}$	4,5
7.	$\{\neg r(b,d,f(f(v7)))\}$	1,6
8.	$\{\neg r(b,y8,f(f(f(v8)))),\neg r(y8,d,f(f(f(v8))))\}$	4,7
9.	$\{\neg r(c,d,f(f(f(v9))))\}$	2,8
10.	$\{\}$	3,9

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Problem Without Repeated Renaming

1.	$\{r(a,b,u1)\}$	Premise
2.	$\{r(b,c,u2)\}$	Premise
3.	$\{r(c,d,u3)\}$	Premise
4.	$\{r(x,z,f(v)),\neg r(x,y,f(f(v))),\neg r(y,z,f(f(v)))\}$	Premise
5.	$\{\neg r(a,d,w)\}$	Goal
6.	$\{\neg r(a,y,f(f(v))),\neg r(y,d,f(f(v)))\}$	4,5
7.	$\{\neg r(b,d,f(f(v)))\}$	1,6
8.	Failure	4,7

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Problem

$$\frac{\{p(x), p(y)\} \quad \{\neg p(u),\neg p(v)\}}{\begin{array}{l}\{p(y),\neg p(v)\}\\ \{p(x),\neg p(v)\}\\ \{p(y),\neg p(u)\}\\ \{p(x),\neg p(u)\}\end{array}}$$

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Factors

If a subset of the literals in a clause Φ has a most general unifier γ, then the clause Φ' obtained by applying γ to Φ is called a *factor* of Φ.

Clause

$$\{p(x),p(f(y)),r(x,y)\}$$

Factors

$$\{p(f(y)),r(f(y),y)\}$$

$$\{p(x),p(f(y)),r(x,y)\}$$

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Relational Resolution III (Final Version)

Φ

Ψ

$((\Phi' - \{\phi\})\tau \cup (\Psi' - \{\neg\psi\}))\sigma$

where $\phi \in \Phi'$, a factor of Φ

where $\neg\psi \in \Psi'$, a factor of Ψ

where $\sigma = mgu(\varphi\tau, \psi)$

where τ is a variable renaming on φ

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Example

$\{p(x), p(y)\}$
$\{\neg p(u), \neg p(v)\}$
$\{p(y), \neg p(v)\}$
$\{p(x), \neg p(v)\}$
$\{p(y), \neg p(u)\}$
$\{p(x), \neg p(u)\}$

$\{p(x)\}$
$\{\neg p(u)\}$
$\{\}$

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Need for Original Clauses

1. $\{p(a,y), p(x,b)\}$	Premise
2. $\{\neg p(a,d)\}$	Premise
3. $\{\neg p(c,b)\}$	Premise
4. $\{p(x,b)\}$	1,2
5. $\{\}$	3,4

1. $\{p(a,y), p(x,b)\}$	Premise
2. $\{\neg p(a,d)\}$	Premise
3. $\{\neg p(c,b)\}$	Premise
4. $\{p(a,b)\}$	Factor of 1

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Provability

A *resolution derivation* of a clause φ from a set Δ of clauses is a sequence of clauses terminating in φ in which each item is
(1) a member of Δ or
(2) the result of applying the resolution to earlier items.

A sentence φ is *provable* from a set of sentences Δ by resolution if and only if there is a derivation of the empty clause from the clausal form of $\Delta \cup \{\neg\varphi\}$.

A resolution *proof* is a derivation of the empty clause from the clausal form of the premises and the negation of the desired conclusion.

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Soundness and Completeness

Metatheorem: Provability using the Relational Resolution Principle is sound and complete for Relational Logic (without equality).

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