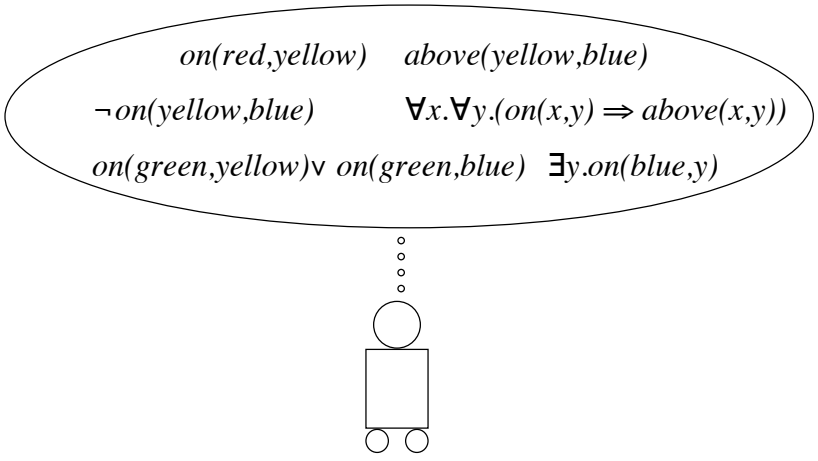
Computational Logic Lecture 1

Introduction

Michael Genesereth Autumn 2008

Computational Logic



2

Mathematics

Group Axioms

$(x \times y) \times z = x \times (y \times z)$

$x \times e = x$

$e \times x = x$

$x \times x^{-1} = e$

Theorem

$x^{-1} \times x = e$

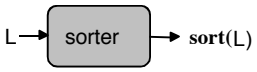
Tasks:

Proof Checking
Proof Generation

3

Software Engineering

Program



Specification:

$$\forall i.\forall j.(i < j \Rightarrow sort(L)_i < sort(L)_j)$$

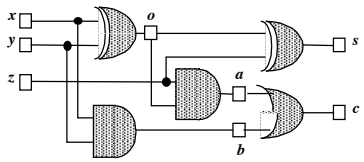
Tasks:

Partial Evaluation
Program Verification
Proof of Termination
Complexity Analysis

4

Hardware Engineering

Circuit:



Behavior:

$o \Leftrightarrow (x \wedge \neg y) \vee (\neg x \wedge y)$

$a \Leftrightarrow z \wedge o$

$b \Leftrightarrow x \wedge y$

$s \Leftrightarrow (o \wedge \neg z) \vee (\neg o \wedge z)$

$c \Leftrightarrow a \vee b$

Applications:

- Simulation
- Configuration
- Diagnosis
- Test Generation

5

Constraint Satisfaction Systems

	6		1		4		5	
		8	3		5	6		
2								1
8			4		7			6
		6				3		
7			9		1			4
5								2
		7	2		6	9		
	4		5		8		7	

6

Deductive Database Systems

Database Tables

parent

art	*bob*
art	*bea*
bea	*coe*

parent(*art*,*bob*)
parent(*art*,*bea*)
parent(*bob*,*coe*)

Queries

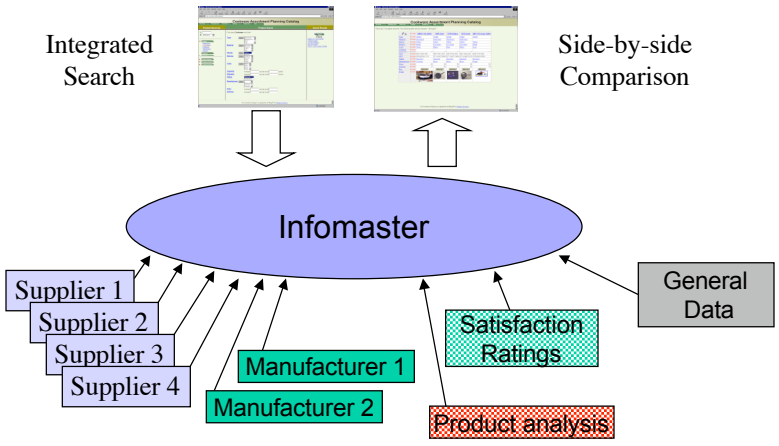
query(X,Z) :- parent(X,Y) & parent(Y,Z)

Constraints

illegal :- parent(X,X)
illegal :- parent(X,Y) & parent(Y,X)

7

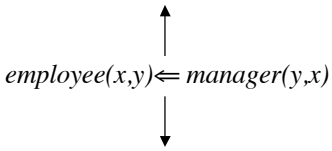
Data Integration



8

Data Integration

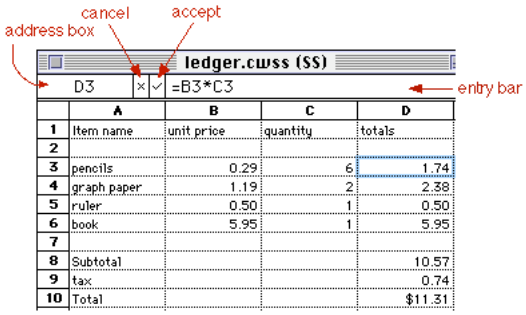
name	employee	location	telephone
John		MJH222	7238086
Jane		Cedar12	7257493
Jill	John	MJH222	
Jerry	Jane	420-032	7256777



name	manager	office	phone
John	Jill	MJH222	38086
Jane	Jerry	Cedar12	57493
Jill		MJH222	
Jerry		420-032	56777

9

Logical Spreadsheets



	A	B	C	D
1	Item name	unit price	quantity	totals
2				
3	pencils	0.29	6	1.74
4	graph paper	1.19	2	2.38
5	ruler	0.50	1	0.50
6	book	5.95	1	5.95
7				
8	Subtotal			10.57
9	tax			0.74
10	Total			$11.31

10

Examples of Non-Functional Constraints

Scheduling

- –Start times must be before end times
- –Room 104 may not be scheduled after 5:00 pm
- –Only senior managers can reserve the third floor conference room

Travel Reservations

- –The number of lap infants in a group on a flight must not exceed the number of adults.

Academic Programs

- –Students must take at least 2 math courses

11

Regulations and Business Rules

Using the language of logic, it is possible to define new relations.

Office mates are people who share an office.

```
officemate(X,Y) :- office(X,Z) ∧ office(Y,Z)
```

This includes the property of legality / illegality.

Managers and subordinates may not be office mates.

```
illegal :- manages(X,Y) ∧ officemate(X,Y)
```

12

Michigan Lease Termination Clause

The University may terminate this lease when the Lessee, having made application and executed this lease in advance of enrollment, is not eligible to enroll or fails to enroll in the University or leaves the University at any time prior to the expiration of this lease, or for violation of any provisions of this lease, or for violation of any University regulation relative to resident Halls, or for health reasons, by providing the student with written notice of this termination 30 days prior to the effective data of termination; unless life, limb, or property would be jeopardized, the Lessee engages in the sales of purchase of controlled substances in violation of federal, state or local law, or the Lessee is no longer enrolled as a student, or the Lessee engages in the use or possession of firearms, explosives, inflammable liquids, fireworks, or other dangerous weapons within the building, or turns in a false alarm, in which cases a maximum of 24 hours notice would be sufficient.

13

Logical Version

$A \Leftarrow A1 \wedge A2 \wedge \neg B$
$A \Leftarrow A4 \wedge \neg B$
$A \Leftarrow A5 \wedge \neg B$
$A \Leftarrow A6 \wedge \neg B$
$A \Leftarrow A7 \wedge \neg B$

$B \Leftarrow B1$
$B \Leftarrow B2$
$B \Leftarrow B3$
$B \Leftarrow B4$
$B \Leftarrow B5$

14

Elements

15

Formal Mathematics

Algebra

1. Formal language for encoding information
2. Legal transformations
3. Automation

Logic

1. Formal language for encoding information
2. Legal transformations
3. Automation

16

Algebra Problem

Xavier is three times as old as Yolanda. Xavier's age and Yolanda's age add up to twelve. How old are Xavier and Yolanda?

17

Algebra Solution

Xavier is three times as old as Yolanda. Xavier's age and Yolanda's age add up to twelve. How old are Xavier and Yolanda?

$$\begin{aligned} x - 3y &= 0 \\ x + y &= 12 \\ \hline -4y &= -12 \end{aligned}$$

$$\begin{aligned} y &= 3 \\ x &= 9 \end{aligned}$$

Automation: Saint, Sin, Reduce, Macsyma, Mathematica

18

Logic Problem

If Mary loves Pat, then Mary loves Quincy. If it is Monday, then Mary loves Pat or Quincy. If it is Monday, does Mary love Quincy?

If it is Monday, does Mary love Pat?

Mary loves only one person at a time. If it is Monday, does Mary love Pat?

19

Formalization

Simple Sentences:

Mary loves Pat.	p
Mary loves Quincy.	q
It is Monday.	m

Premises:

If Mary loves pat, Mary loves Quincy.	$p \Rightarrow q$
If it Monday, Mary loves Pat or Quincy.	$m \Rightarrow p \vee q$
Mary loves one person at a time.	$p \wedge q \Rightarrow$

Questions:

Does Mary love Pat?	$\Rightarrow p$
Does Mary love Qunicy?	$\Rightarrow q$

20

Rule of Inference

Propositional Resolution

$$\frac{\begin{array}{cc} p_1 \wedge \ldots \wedge p_k & \Rightarrow \quad q_1 \vee \ldots \vee q_l \\ r_1 \wedge \ldots \wedge r_m & \Rightarrow \quad s_1 \vee \ldots \vee s_n \end{array}}{p_1 \wedge \ldots \wedge p_k \wedge r_1 \wedge \ldots \wedge r_m \quad \Rightarrow \quad q_1 \vee \ldots \vee q_l \vee s_1 \vee \ldots \vee s_n}$$

NB: If p_i is the same as s_j, it is okay to drop the two symbols, with the proviso that *only one* such pair may be dropped.

NB: If a constant is repeated on the left or the right, all but one of the occurrences can be deleted.

21

Examples

$$\frac{\begin{array}{cc} p & \Rightarrow \quad q \\ & \Rightarrow \quad p \end{array}}{\Rightarrow \quad q} \qquad \frac{\begin{array}{cc} p & \Rightarrow \quad q \\ q & \Rightarrow \end{array}}{p \quad \Rightarrow} \qquad \frac{\begin{array}{cc} p & \Rightarrow \quad q \\ q & \Rightarrow \quad r \end{array}}{p \quad \Rightarrow \quad r}$$

22

Logic Problem Revisited

If Mary loves Pat, then Mary loves Quincy. If it is Monday, then Mary loves Pat or Quincy. If it is Monday, does Mary love Quincy?

$$\begin{array}{lcl} p & \Rightarrow & q \\ m & \Rightarrow & p \vee q \\ \hline m & \Rightarrow & q \vee q \\ m & \Rightarrow & q \end{array}$$

23

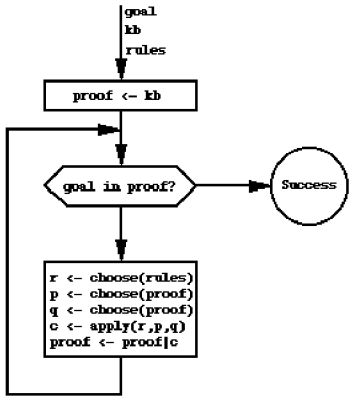
Logic Problem Concluded

Mary loves only one person at a time. If it is Monday, does Mary love Pat?

$$\begin{array}{lcl} m & \Rightarrow & q \\ p \wedge q & \Rightarrow & \\ \hline m \wedge p & \Rightarrow & \end{array}$$

24

Automated Reasoning



25

Logic Technology

Languages
- Knowledge Interchange Format (KIF) - ANSI
- Common Logic - W3C

Some Popular Automated Reasoning Systems
- Otter
- PTTP / Epilog
- Snark / Vampire / …

Knowledge Bases
- Definitions (*Bachelor is an unmarried adult male*.)
- Constraints (e.g. *PV=nRT*)
- Laws (e.g. *1040 necessary if earnings* > $10,000.)

26

Study Guide

27

Multiple Logics

Propositional Logic

If it is raining, the ground is wet.

Relational Logic

If x is a parent of y, then y is a child of x.

28

Common Topics

Common Topics
- Syntax - expressions in the language
- Semantics - meaning of expressions
- Computational Matters

Contrasts
- Expressiveness - operators, variables, terms, ...
- Computational Hierarchy - polynomial? decidable?
- Tradeoffs - expressiveness versus computability

29

Meta

We will frequently write sentences *about* sentences.

Sentence: *When it rains, it pours*.
Metasentence: *That sentence contains two verbs*.

We will frequently prove things about proofs.

Proofs: formal
Metaproofs: informal

30

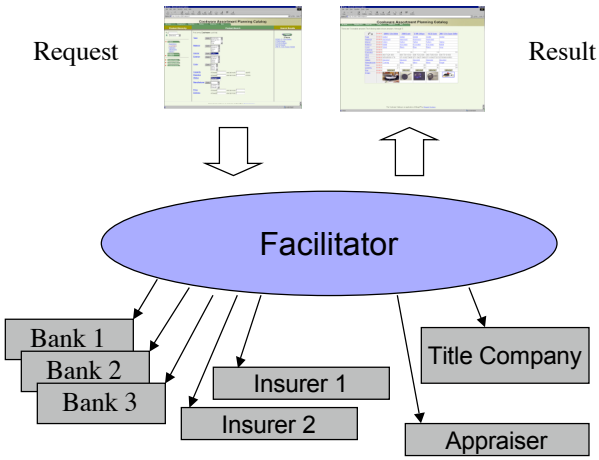
Mike took it twice!

31

http://cs157.stanford.edu

32

Web Service Integration



QBF (Query by Form) - User queries Internet in his own schema
ABF (Application by Form) - User requests services using own schema

33